Approximate Method for Determining the Stability of the Film-Casting Process

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Determination of the stability of polymer processing is always a major concern to people in both industry and academia. This is because the productivity issue, including product quality control, is of primary importance to the former group, while to the latter the existence of stable solutions and process sensitivity to outside disturbances are usually of fundamental interest. The continuous processes of polymer processing like film casting, fiber spinning, and film blowing are of course no exception.

While the issue of stability in the whole range of nonlinear dynamics is vast, it takes a lot of effort to determine its entirety. In most industrial processes where there are not many desirable stable solutions, simple linear stability is usually what is needed. For this end, linear stability analysis, which consists of linearization, variables perturbation, and solving the eigenvalue matrix equation, is well established in engineering fields as the method for determining the stable operational regions for the involved processes.

There are, however, two aspects in this linear stability analysis where some improvements are often desirable. One is simplification of the computations, if possible, and the other is how to maintain an understanding of the physics of the system, which is quite often lost while numerically computing the eigenvalues.

In this study, it is intended to make these two improvements in the stability analysis of the film-casting process by developing a good approximate method based on the fundamental physics of the system. The example process considered is that of Anturkar and Co (1988), who have thoroughly studied the case and reported exact stability results with great accuracy and detail.

Using the stability criterion derived for fiber spinning by Kim et al. (1996), an approximate stability criterion composed of the traveling times of throughput waves and the fluid residence time has been developed. A way of computing these traveling times using only steady-state solutions was then devised. The stability curves thus obtained using this method are seen to agree well with the exact ones.

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Derivation of an Approximate Criterion for Draw Resonance

The instability criterion for isothermal spinning of Newtonian fluids, or draw resonance, was given by Kim et al. (1996), as below.

$$2t_L + \frac{T}{2} \leq 2\theta_L = T + 2(\Delta t) \quad \text{for} \quad r \geq r_c, \tag{1}$$

where t_L = dimensionless traveling time of throughput waves; T = dimensionless period of draw resonance; θ_L = dimensionless traveling time of maximum and minimum cross-sectional area waves; Δt = dimensionless time (phase) difference between the spinline force and the cross-sectional area wave at the takeup; r = drawdown ratio; and r_c = critical drawdown ratio at the onset of draw resonance.

Next, Eq. 1 is rewritten at the onset point of draw resonance as follows:

$$2t_L + \Delta t = \frac{T}{2} + 3(\Delta t) \quad \text{at} \quad r = r_c. \tag{2}$$

The reason for writing Eq. 2 this way is because it turns out that the times shown on both sides of the equation are approximately equal to the fluid residence time. This is easily seen from the various times of the system of isothermal spinning of Newtonian fluids at the onset of draw resonance, that is, $t_L = 0.1419$; $\Delta t = 0.0298$; $T = 0.4484$; $\theta_L = 0.2538$; and $\tau_L = 0.3162$ at $r = r_c = 20.218$ (Kim et al., 1996).

This approximate relation among the times of the system also holds for the isothermal spinning of convected Maxwell fluids. For example, the system of Hyun (1978) with a Deborah number (or the dimensionless relaxation time) of 0.005 yields the following data set (Jung and Hyun, 1999): $t_L = 0.1268$; $\Delta t = 0.0263$; $T = 0.4020$; $\theta_L = 0.2273$; and $\tau_L = 0.2767$ at $r = r_c = 25.45$. One thing to notice here is that the accuracy of the approximation for the Newtonian case is better than that of the viscoelastic case: that is, the error between the fluid residence time and the time of Eq. 2 is 0.82% (Newtonian case) and 1.16% (viscoelastic case).

An approximate criterion for draw resonance is now in our possession:

$$2\left(t_L + \frac{\Delta t}{2}\right) \approx \tau_L \quad \text{at} \quad r = r_c, \tag{3}$$

where

$$t_L = \int_0^1 \frac{dx}{U} \tag{4}$$

$$\tau_L = \int_0^1 \frac{dx}{v}, \tag{5}$$

and τ_L = fluid residence time; U = dimensionless traveling velocity of throughput waves; v = dimensionless spinline velocity; x = dimensionless spinning distance from the spinneret.

It is noted here that the computation on the righthand side of Eq. 3 is rather straightforward via Eq. 5 using the steady-state velocity solution, which is easily obtained. The computation of U and that of the lefthand side of Eq. 3 are, on the other hand, quite involved, because the value of a difficult partial derivative needs to be obtained, as shown below. So now we will see whether there is a way to obtain the values of U and t_L in a simple, albeit approximate, manner without many numerical computations.

The exact expression for U in Eq. 4 was derived by Hyun (1978):

$$U \equiv \left[\frac{\partial(Av)}{\partial A}\right]_x = -\frac{A\left(\dfrac{\partial v}{\partial x}\right)_A}{\left(\dfrac{\partial A}{\partial x}\right)_{Av}}, \tag{6}$$

where A = dimensionless spinline cross-sectional area.

While in Eq. 6 the value of the denominator is easily obtainable from the steady-state solution, a great deal of numerical computation is needed for the numerator because of the partial derivative involved there. This particular partial derivative is numerically computed following the steps shown below:

$$\left(\frac{\partial v}{\partial x}\right)_A = \left(\frac{\partial v}{\partial x}\right)_t + \left(\frac{\partial v}{\partial t}\right)_x \left(\frac{\partial t}{\partial x}\right)_A$$

$$= \left(\frac{\partial v}{\partial x}\right)_t - \left(\frac{\partial v}{\partial t}\right)_x \frac{\left(\dfrac{\partial A}{\partial x}\right)_t}{\left(\dfrac{\partial A}{\partial t}\right)_x}. \tag{7}$$

Here the partial derivative has been transformed from the (x, A) coordinate system to the (x, t) one. The presence of $(\partial A/\partial t)_x$ in the denominator would have caused a problem if we had tried to perform computations at a fixed position of x, because $(\partial A/\partial t)_x$ vanishes periodically in draw resonance. However, it won't happen here because the traveling throughput wave is followed in computing the partial derivatives. Actually the throughput wave with the constant value of unity is tracked, because only these unity–throughput waves travel the entire spinning distance from the spinneret to the takeup due to the unity–throughput boundary condition maintained at the spinneret, while other nonunity waves travel only some portion of the spinning distance (Kim et al., 1996).

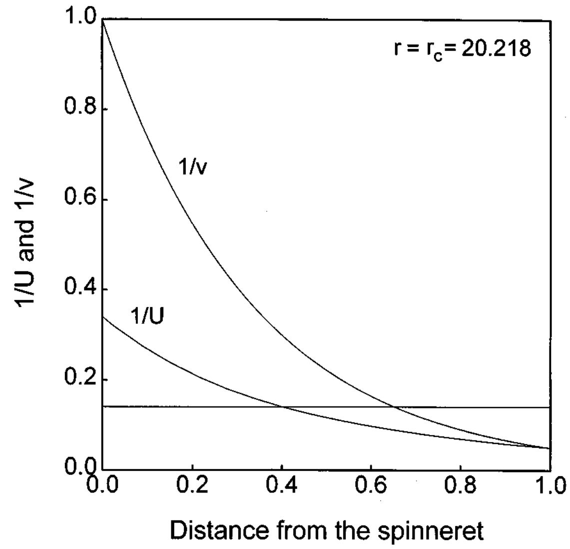

Figure 1. Reciprocal velocity of throughput waves and the reciprocal spinline velocity.

Once the value of U has been numerically obtained, which is the same whether it is being computed according to Eqs. 6 and 7 or by simply following the throughput wave with the magnitude of unity, the wave traveling time of Eq. 4 becomes the area under the curve of $1/U$ plotted against x. Corresponding to this area, a rectangle with an equal area was found, as shown in Figure 1. The next task is then how to determine the height of this rectangle.

Neglecting $(\Delta t/2)$, which is small as compared to t_L, computation of the lefthand side of Eq. 3 thus boils down to finding the appropriate height of this rectangle, which is equivalent to finding the average velocity of the throughput waves, which yields the same traveling time as the one given by Eq. 4. For this purpose, the approximate expression derived by Hyun (1978) for Newtonian, power-law, and Maxwell fluids as contained in the expression for t_L is used here:

$$U_{\text{av}} = \frac{r-1}{\ln r}. \tag{8}$$

Then, by neglecting Δt in Eq. 3 and replacing t_L by $1/U_{\text{av}}$, the following approximate criterion for draw resonance is obtained:

$$2\left(\frac{\ln r}{r-1}\right) \approx \tau_L. \tag{9}$$

The errors involved in using Eq. 9 for the Newtonian and viscoelastic fluids cases mentioned after Eq. 2 have been found to be 1.04% and 4.30%, respectively. As with the earlier finding, the Newtonian case has a better accuracy than the viscoelastic one.

Equation 9 is thus the equation that can be used to approximately determine the draw resonance stability of such extensional deformation processes as film casting, film blowing, and fiber spinning.

Stability Results of a Film-Casting Process

The example process to which the approximate method of Eq. 9 is applied is the film-casting process, which has been analyzed with great accuracy and detail by Auturkar and Co (1988). They reported exact stability results for this process where they assumed the width of the film did not change from die exit to the chill roll. Their governing equations are presented here only for the purpose of reference, while the reader is referred to the original source for other details.

Equation of continuity:

$$\frac{\partial H}{\partial \theta} + \frac{\partial(\phi_\xi H)}{\partial \xi} + \frac{\partial(\phi_\zeta H)}{\partial \zeta} = 0, \tag{10}$$

where $\phi_i = v_i/v_o$, $H = h/h_o$.

Equation of motion:

$$\frac{\partial\left[(T_{\xi\xi} - T_{\chi\chi})H\right]}{\partial \xi} + \frac{\partial(T_{\xi\zeta} H)}{\partial \zeta} = 0,$$

$$\frac{\partial(T_{\xi\zeta} H)}{\partial \xi} + \frac{\partial\left[(T_{\zeta\zeta} - T_{\chi\chi})H\right]}{\partial \zeta} = 0, \tag{11}$$

where $T_{ij} = \tau_{ij} L/\eta_o v_o$.

Constitutive equation (a modified convected Maxwell model):

$$T_{\chi\chi} + \Lambda_o \Lambda\left[\frac{\partial T_{\chi\chi}}{\partial \theta} + \phi_\xi \cdot \frac{\partial T_{\chi\chi}}{\partial \xi} + \phi_\zeta \cdot \frac{\partial T_{\chi\chi}}{\partial \zeta} + 2T_{\chi\chi}\left(\frac{\partial \phi_\xi}{\partial \xi} + \frac{\partial \phi_\zeta}{\partial \zeta}\right)\right] = 2\Psi\left(\frac{\partial \phi_\xi}{\partial \xi} + \frac{\partial \phi_\zeta}{\partial \zeta}\right)$$

$$T_{\xi\xi} + \Lambda_o \Lambda\left[\frac{\partial T_{\xi\xi}}{\partial \theta} + \phi_\xi \cdot \frac{\partial T_{\xi\xi}}{\partial \xi} + \phi_\zeta \cdot \frac{\partial T_{\xi\xi}}{\partial \zeta} - 2T_{\xi\xi} \cdot \frac{\partial \phi_\xi}{\partial \xi} - 2T_{\xi\zeta} \cdot \frac{\partial \phi_\xi}{\partial \zeta}\right] = -2\Psi\frac{\partial \phi_\xi}{\partial \xi}$$

$$T_{\zeta\zeta} + \Lambda_o \Lambda\left[\frac{\partial T_{\zeta\zeta}}{\partial \theta} + \phi_\xi \cdot \frac{\partial T_{\zeta\zeta}}{\partial \xi} + \phi_\zeta \cdot \frac{\partial T_{\zeta\zeta}}{\partial \zeta} - 2T_{\zeta\zeta} \cdot \frac{\partial \phi_\zeta}{\partial \zeta} - 2T_{\xi\zeta} \cdot \frac{\partial \phi_\zeta}{\partial \xi}\right] = -2\Psi\frac{\partial \phi_\zeta}{\partial \zeta}$$

$$T_{\xi\zeta} + \Lambda_o \Lambda\left[\frac{\partial T_{\xi\zeta}}{\partial \theta} + \phi_\xi \cdot \frac{\partial T_{\xi\zeta}}{\partial \xi} + \phi_\zeta \cdot \frac{\partial T_{\xi\zeta}}{\partial \zeta} - T_{\xi\zeta}\left(\frac{\partial \phi_\xi}{\partial \xi} + \frac{\partial \phi_\zeta}{\partial \zeta}\right) - T_{\xi\xi} \cdot \frac{\partial \phi_\zeta}{\partial \xi} - T_{\zeta\zeta} \cdot \frac{\partial \phi_\xi}{\partial \zeta}\right] = -\Psi\left(\frac{\partial \phi_\zeta}{\partial \xi} + \frac{\partial \phi_\xi}{\partial \zeta}\right), \tag{12}$$

where $\Psi = \eta/\eta_o$, $\Lambda = \lambda/\lambda_o$, and $\Lambda_i = \lambda_i v_o/L$.

Carreau viscosity function and fluid characteristic time function:

$$\Psi = \left(1 + \Lambda_v^2 \dot{\Gamma}^2\right)^{(n-1)/2}, \qquad \Lambda = \left(1 + \Lambda_t^2 \dot{\Gamma}^2\right)^{(n'-1)/2}. \tag{13}$$

The notation appearing in these equations is as follows: h = film thickness; H = dimensionless film thickness; τ_{ij} = components of stress tensor; T_{ij} = components of dimensionless stress tensor; θ = dimensionless time; χ = dimensionless film-thickness coordinate; ξ = dimensionless film-width coordinate; ζ = dimensionless axial coordinate; ϕ_i = components of the dimensionless velocity vector; η = viscosity; η_o = zero-shear-rate viscosity; λ = fluid characteristic time; λ_o = fluid characteristic time at zero-shear-rate; Ψ = dimensionless viscosity; Λ = dimensionless fluid characteristic time; and Λ_o, Λ_t, Λ_v = dimensionless time constants corresponding to λ_o, λ_t, λ_v, respectively. (Note that some of these notations inevitably coincide with those that appeared earlier in our study, but of course they have different meanings here.)

Following the usual steps of linear stability analysis, Anturkar and Co linearized these nondimensionalized governing equations, introduced perturbed dependent variables, and then solved the stability eigenvalue matrix equation. By finding the values of the drawdown ratio that yield zero for the real parts of the largest eigenvalues at the onset of draw resonance, they obtained the stability curves of the system.

Figure 2 compares the exact stability results thus obtained by Anturkar and Co (1988) with the approximate ones obtained by us when we applied the method of Eq. 9 to the case that showed the effect of the exponent n of the viscosity function of Eq. 13 (Choi, 1992).

Despite the approximations introduced in the course of deriving Eq. 9, the general shapes of the curves produced by the approximate method are quite similar to those obtained by using the exact method. The quantitative agreement between the exact and approximate curves, however, is not

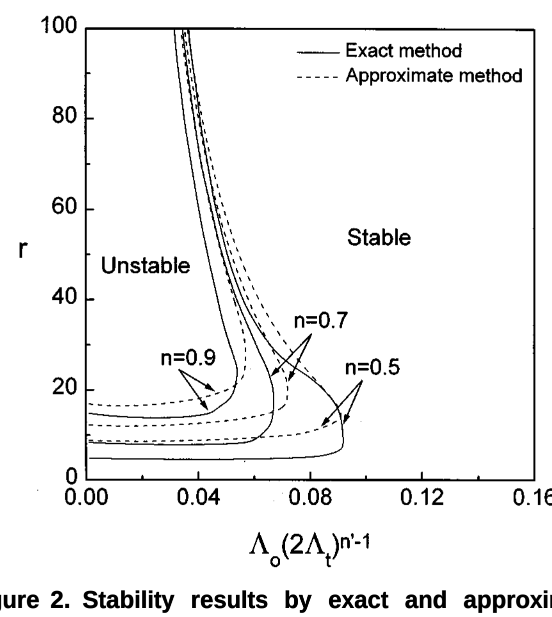

Figure 2. Stability results by exact and approximate methods for $\Lambda_t = \Lambda_v = 100$ and $n' = 0.7$.

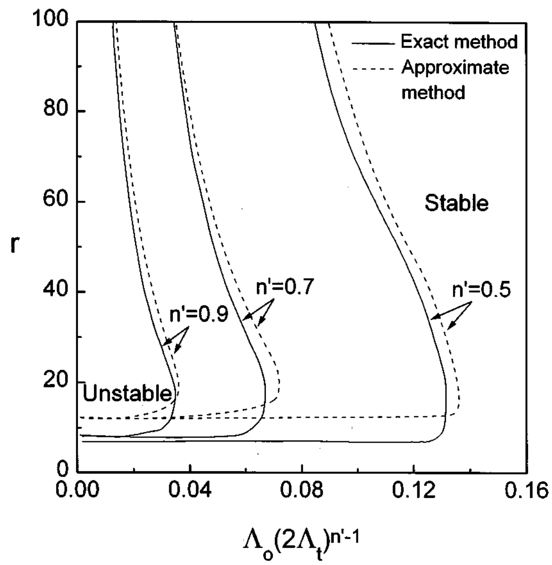

Figure 3. Stability results by exact and approximate methods for $\Lambda_t = \Lambda_v = 100$ and $n = 0.7$.

necessarily good at all times. Also, as the shear thinning increases (i.e., as n decreases), the discrepancies increase, which agrees with the earlier findings after Eqs. 2 and 9, where accuracy for the viscoelastic fluids is not as good as for the Newtonian fluids. Summarizing the utility of the approximate method in this study, we need to mention three points. First, our method uses only the steady-state velocity solution of the system in finding the stability curves, and thus it is extremely simple and fast. Second, in determining the stability of the film-casting process, the fundamental physics of the system has been utilized: the comparison of the traveling times of the kinematic waves of the system determines the stability. Third, it can be said that the fundamental nature of draw resonance in film casting is very like that in fiber spinning.

Figure 3 shows another comparison of the stability results by the exact and approximate methods, respectively, illustrating the effect of another exponent, n', of the characteristic time function of Eq. 13. Again, just as with Figure 2, the qualitative agreement between the exact and approximate methods is good, whereas the quantitative agreement gets better as the value of n' approaches unity.

Conclusions

A simple, approximate method for determining the stability of film casting has been developed that utilizes the same concept of traveling throughput waves that was previously used to derive the draw resonance criterion in fiber spinning (Kim et al., 1996). Applying this approximate method to the film-casting process of Anturkar and Co (1988), stability curves were obtained that are qualitatively quite close to the exact ones. The significance of this finding is threefold. First, the fundamental nature of draw resonance in film casting is very similar to that in fiber spinning. Second, the approximate method developed in this study requires only the steady-state solution of the system, which results in the rapid determination of stability as compared to the exact stability analysis, which requires solving transient equations. Third, the physics behind the film-casting process is fully utilized in the approximate method, in contrast to the exact stability method, where the physics of the system is usually lost while computing the eigenvalues. It was also found that the accuracy of the approximate method decreases as the non-Newtonian nature of fluids increases. This approximate stability-determining method can be applied equally well as a useful analysis tool to other extensional deformation processes, including other film-casting systems such as the one by Silagy et al. (1996).

Acknowledgments

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