Draw Resonance and Kinematic Waves in Viscoelastic Isothermal Spinning

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Draw resonance, one of major instabilities in polymer processing, arises as the drawdown ratio is increased beyond its critical value and is manifested by sustained periodic variations in spinline variables such as cross-sectional area and tension. Ever since Christensen (1962) and Miller (1963) first discovered the phenomenon and aptly named it as such, draw resonance has been the subject of research for many people around the world since it is closely related to the industrially important productivity issue and the academically interesting stability topic.

Thanks to those research efforts in the past four decades, draw resonance is now fairly well understood. See, for example, Pearson and Matovich (1969), Gelder (1971), Ishihara and Kase (1976), Fisher and Denn (1976), White and Ide (1978), Hyun (1978), Kase and Araki (1982), Schultz et al. (1984), Beris and Liu (1988), Bechtel et al. (1992), Gupta et al. (1996), and Silagy et al. (1996, 1998). First, draw resonance, a supercritical Hopf bifurcation, is a hydrodynamic instability, not a viscoelastic one, albeit altered by fluid viscoelasticity, and, thus, even Newtonian fluids can exhibit it. Second, its onset is readily and accurately predicted employing linear stability analysis if the governing equations are provided. However, the physics behind this draw resonance as to why and how it occurs has not been fully understood.

As Petrie (1988) eloquently expounded, there are still many topics and issues to be answered surrounding draw resonance. In an effort to shed light on this instability, a new concept was tried focusing on the hyperbolic nature of the system, that is, kinematic waves traveling along the spinline (Hyun, 1978; Kim et al., 1996; Jung et al., 1999a). Specifically, comparing the traveling times of throughput waves and those of maximum and minimum spinline cross-sectional area waves, a new criterion for draw resonance was derived. As the drawdown ratio increases, this stability criterion determines its critical value at the onset point exactly agreeing with the results by the linear stability analysis.

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The significance of this criterion for draw resonance is illustrated below. Unlike the linear stability analysis which doesn't say much about the physics of the system behind the phenomenon other than numerically computing the eigenvalues determining the growth rate of disturbances, this new concept explains why and how draw resonance originates and persists. The fundamental mechanism behind the ingenious device called a draw resonance eliminator pioneered and successfully implemented by Union Carbide researchers (Lucchesi, Roberts, and Kurtz, 1985) can be explained using the concept, as shown by Jung et al. (1999b). Also, based on the concept, Jung et al. (1999a) derived an approximate method for determining the stability of the film casting case on which Anturkar and Co (1988) carried out an excellent stability analysis.

In this article the same concept is applied to the case of viscoelastic spinning adopting an upper convected Maxwell model, a simple yet widely used constitutive relation in modeling various polymer processing. Differences between the Newtonian and viscoelastic fluids spinnings are also discussed as related to their stability.

Description of the System

The isothermal melt spinning of an upper convected Maxwell fluid has the following governing equations (Jung and Hyun, 1999).

Continuity equation

$$\partial A/\partial t + \partial(AV)/\partial x = 0 \tag{1}$$

where $A = a/a_0$, $V = v/v_0$, $t = t' v_0/L$, $x = z/L$.

Equation of motion

$$\partial(A\tau)/\partial x = 0 \tag{2}$$

where $\tau = \sigma L/2\eta_0 v_0$.

Constitutive equation

$$\tau\left(1+\bar{a}\sqrt{3}\,De(\partial V/\partial x)\right)+De\big((\partial\tau/\partial t)$$
$$+V(\partial\tau/\partial x)-2\tau(\partial V/\partial x)\big)=(\partial V/\partial x) \quad (3)$$

where $\lambda=\lambda_0/(1+\bar{a}\sqrt{3}\,\mathrm{De}(\partial V/\partial x))$, $\quad De=\lambda_0 v_0/L$

These dimensionless equations are subject to the following boundary conditions

$$\begin{aligned} t=0:&\ A=A_s,\ V=V_s,\ \tau=\tau_s && \text{for } 0<x<1 \\ t>0:&\ A=A_0=1,\ V=V_0=1 && \text{at } x=0 \\ &\ V=V_L=r(1+\epsilon) && \text{at } x=1 \end{aligned} \quad (4)$$

where a is the spinline cross-sectional area, A is the dimensionless a, v is the spinline velocity, V is dimensionless v, σ is the spinline axial stress, τ is the dimensionless σ, z is the distance coordinate from spinneret, x is the dimensionless z, $t'=$ time, $t=$ dimensionless t', De is the Deborah number, $\bar{a}$ is the parameter, η_0 is the zero shear rate viscosity, λ is the material relaxation time, λ_0 is λ at zero extension rate, r is the drawdown ratio, ϵ is the constant representing the initial disturbance at the takeup, and subscripts 0, L, S denote spinneret, takeup, and steady-state conditions, respectively.

The parameter $\bar{a}$ in Eq. 3 represents the strain rate dependency of material relaxation time which was introduced by White and Ide (1978) to properly portray different extensional characteristics of such polymers as LDPE, HDPE, LLDPE, and PP, and successfully applied by Minoshima and White (1986) to the analysis of various extensional deformation processes.

In the above, several assumptions have been incorporated. First, the variations of variables across the spinline cross-section are neglected to result in an one-dimensional model. Second, the origin of the distance coordinate is chosen at the die (extrudate) swell position ignoring the pre-spinneret conditions on the spinline. Third, all the secondary forces, that is, gravity, air drag, surface tension, and inertia, are neglected. Fourth, in Eq. 3 instead of a full set of equations including the radial stress equation only the axial stress equation is solved as an approximation with the radial stress obtainable by the method of Beris and Liu (1988).

Transient Simulation Results and Criterion for Draw Resonance

The transient simulation results of this viscoelastic fluid spinning are similar to those of Newtonian spinning reported by Kim et al. (1996). To avoid numerical instability due to viscoelasticity at high De, a first-order upwind implicit method has been used. Figure 1 shows the numerical results of a typical viscoelastic case, that is, $\bar{a}=0.4$, $De=0.019$ at $r=r_c=27.97$ in the x-t grid of $2{,}000\times10{,}000$ mesh points which guarantee acceptable accuracy for the determination of critical onset points. The severity of draw resonance is shown to increase with the decreasing mesh size due to the existence of extremely sharp spikes in the curves of draw resonance. The value of 0.4 for parameter $\bar{a}$ was used in this study to represent the extension-thickening behavior of LDPE, as shown by White and Ide (1978). Viscoelastic instability such as Hadamard and dissipative ones didn't arise in our system with the chosen upper convected Maxwell model. The Phan Thien-Tanner model which is supposedly free of the possibility of viscoelastic instability has yielded similar results in our cases. The transient curves (with the y-axis representing the amplitudes of the oscillating spinline variables) of the spinline cross-sectional area (A), velocity (V), throughput (AV), and tension (F) are plotted at five spinline positions (node 1, quarter-distance point, half-distance point, three-quarter-distance point, and takeup), when sufficient time has elapsed after an initial step disturbance ($\epsilon=1\%$) introduced at the takeup velocity. The simulation procedure is that we start from a steady-state solution with a disturbance at the takeup velocity and continue the integration and iteration with convergence of spinline stress and boundary conditions satisfied in each time step. The node 1 was chosen instead of the spinneret ($x=0$) because A, V, and AV are constant at the spinneret due to Eq. 4 (F, however, oscillates with time at all points including the spinneret).

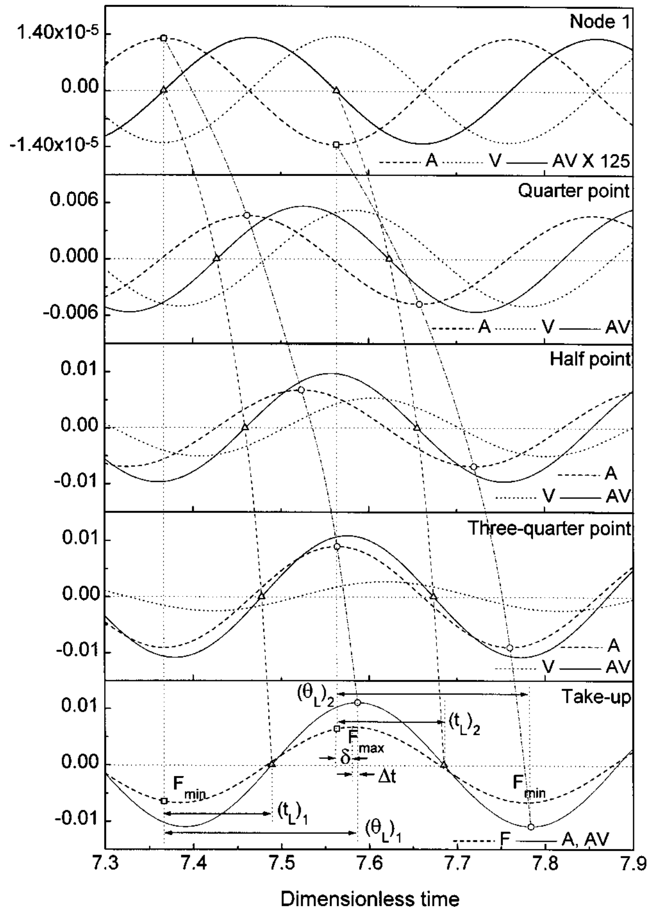

Figure 1. Transient curves of spinline variables at five different spatial points of the spinline when $r=r_c=27.97$ for the case of $\bar{a}=0.4$, $De=0.019$.

The same observations as those in the Newtonian case of Kim et al. (1996) are also pertinent here. (1) Since we are very close to the onset point of draw resonance, all the curves in Figure 1 are almost symmetrical in time; (2) the spinline tension (F) curve is the same at all nodes, that is, independent of x due to Eq. 2 where all the secondary forces were neglected; (3) there are time differences between this tension and the cross-sectional area waves at the takeup; (4) clearly, the waves of $A_{\max}$ and $A_{\min}$ determine the period of the oscillation.

Now, differences between Newtonian and viscoelastic spinning are brought to light. Whereas in the Newtonian case shown by Kim et al. (1996), an $A_{\max}$ wave appears at node 1 at the same moment of the minimum spinline tension $F_{\min}$, in the viscoelastic case shown in Figure 1 an $A_{\max}$ wave appears at node 1 time δ before $F_{\min}$, occurs, that is, there is a time delay for the spinline tension to occur after the corresponding area waves appear at node 1. In other words, unlike in Newtonian spinning where the spinline tension transmits information from the takeup to the spinneret in an instantaneous fashion, in viscoelastic spinning the tension relays the takeup information to the spinneret with a time delay δ. The mechanism of perpetuating oscillations is, however, the same for both Newtonian and viscoelastic cases as illustrated below.

After an $A_{\max}$ wave appears at node 1, it travels down the spinline and exits the system through the takeup. As this $A_{\max}$ wave goes out through the takeup, the spinline tension becomes maximum because the tension is proportional to the area. This maximum tension $F_{\max}$ is not, however, in phase with $A_{\max}$ at the takeup but rather out of phase with it with a time difference Δt in both Newtonian and viscoelastic cases. This maximum tension acting on the entire spinline, in turn, causes a minimum A wave, $A_{\min}$, to appear at node 1 (simultaneously, with $F_{\max}$ for the Newtonian case, but time δ before $F_{\max}$ for viscoelastic case) and to travel toward the takeup. As this $A_{\min}$ passes through the takeup, the spinline tension becomes minimum $F_{\min}$ out of phase with $A_{\min}$ at the takeup with Δt. Then, this minimum tension $F_{\min}$ causes a maximum area $A_{\max}$ to appear at node 1 (simultaneously, with $F_{\min}$ for Newtonian case, but time δ before $F_{\min}$ for the viscoelastic case). The cycle of waves repeats itself in this fashion.

The period of the oscillation in viscoelastic spinning is thus expressed as follows.

$$\begin{aligned} T &= (\text{traveling time of an } A_{\max} \text{ wave} - \Delta t - \delta) \\ &\quad + (\text{traveling time of an } A_{\min} \text{ wave} - \Delta t - \delta) \\ &= (\theta_L - \Delta t - \delta)_1 + (\theta_L - \Delta t - \delta)_2 \end{aligned} \tag{5}$$

There is no time delay between F and A at node 1, that is, $\delta = 0$, for Newtonian case.

Figure 1 also shows that within the same period, two successive unity-throughput waves (AV) are able to travel from the spinneret to the takeup with a pause time of $T/4$ in between. In other words, the following relations are observed.

$$\text{For } r < r_c\text{: } (t_L - \Delta t - \delta)_1 + T/4 + (t_L - \Delta t - \delta)_2 + T/4 > (\theta_L - \Delta t - \delta)_1 + (\theta_L - \Delta t - \delta)_2 = T \tag{6}$$

$$\text{For } r \geq r_c\text{: } (t_L - \Delta t - \delta)_1 + T/4 + (t_L - \Delta t - \delta)_2 + T/4 = (\theta_L - \Delta t - \delta)_1 + (\theta_L - \Delta t - \delta)_2 = T \tag{7}$$

where $(t_L)_1$ and $(t_L)_2$ denote the traveling times of two unity-throughput waves starting from the spinneret time δ before $F_{\min}$ and $F_{\max}$, respectively. Only unity-throughput waves are considered here, because among infinitely many throughput waves only those having the value of unity travel the entire spinning distance, while others travel only portions due to the boundary condition of $AV = 1$ at the spinneret by Eq. 4. Again, in Newtonian cases, $\delta = 0$ in both Eqs. 6 and 7.

It is noted that the two times on the left and right sides of Eq. 7 are equal not only when $r = r_c$, but also when $r > r_c$. This is because when a draw resonance is fully established with a finite amplitude at $r > r_c$, both two unity-throughput waves on the left side and the maximum and minimum A waves on the right side fit into the same single period of oscillation so as to produce steady oscillation.

Now we derive the criterion for draw resonance determining its onset point. Since all the curves are symmetrical at the onset point, as shown in Figure 1, and all time differences of Δt's and δ's are cancelled out from the both sides of Eq. 7, and we have at $r = r_c$

$$2t_L + T/2 = 2\theta_L \tag{8}$$

Utilizing this relation, we can then derive the following criterion for draw resonance

$$(t_L)_1 + (t_L)_2 + T/2 \underset{<}{\overset{>}{=}} (\theta_L)_1 + (\theta_L)_2 \quad \text{for} \quad r \underset{>}{\overset{<}{=}} r_c \tag{9}$$

This criterion is the same as that for Newtonian case because the time delay δ's representing the differences between viscoelastic and Newtonian spinning cases are cancelled out in the equation. The explanation about this criterion for draw resonance is as follows. In order for a steady oscillation of draw resonance to be sustained, two successive unity-throughput waves (left side of Eq. 9) need to travel the spinning distance within the same time period of maximum and minimum A waves (right side of Eq. 9) to travel. For $r < r_c$, the required time for these two unity-throughput waves to travel (left side) is larger than the allowed time by two traveling successive $A_{\max}$ and $A_{\min}$ waves (right side), making the oscillation impossible, that is, the oscillation dies out with time. At $r = r_c$, the times of the both sides become equal making the oscillation just possible. For $r = 40 > r_c$, the required time is smaller than the allowed, so the sustained oscillation is always possible, and transient curves of a typical case are shown in Figure 2.

One comment is worth mentioning here regarding Eq. 7 and relation 9. Whereas Eq. 7 describes the fact that the two successive unity-throughput waves on the left side and $A_{\max}$ and $A_{\min}$ waves on the right side are able to fit into the same period of oscillation to perpetuate draw resonance, that is, the equality holding for $r > r_c$, relation 9 was derived based on the onset conditions of draw resonance so that the inequality holds for $r > r_c$.

Figure 3 depicts the results by the criterion for draw resonance of relation 9, which produces the onset point, agreeing

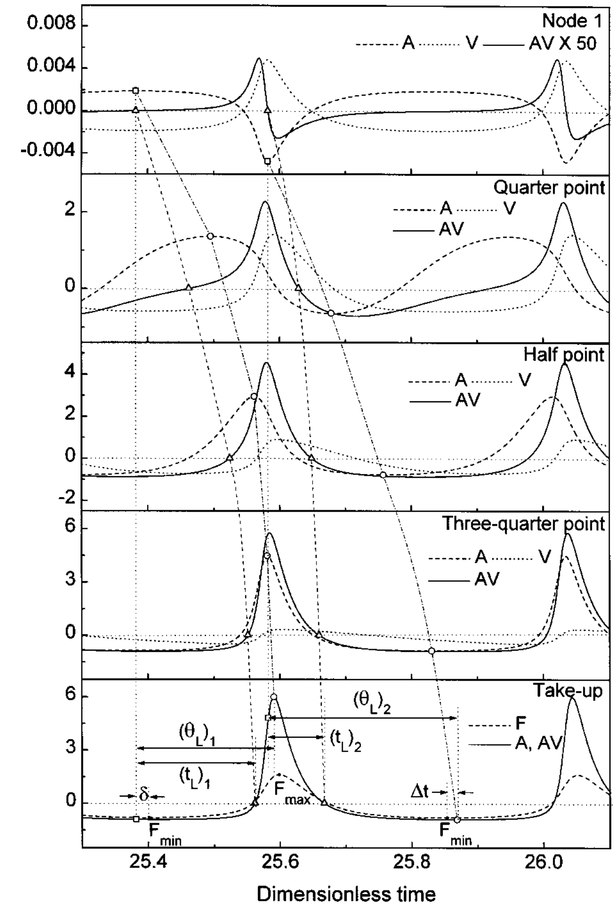

Figure 2. Transient curves of spinline variables at five different spatial points of the spinline when *r* = 40 > r_c for the case of $\bar{a}$ = 0.4, *De* = 0.019.

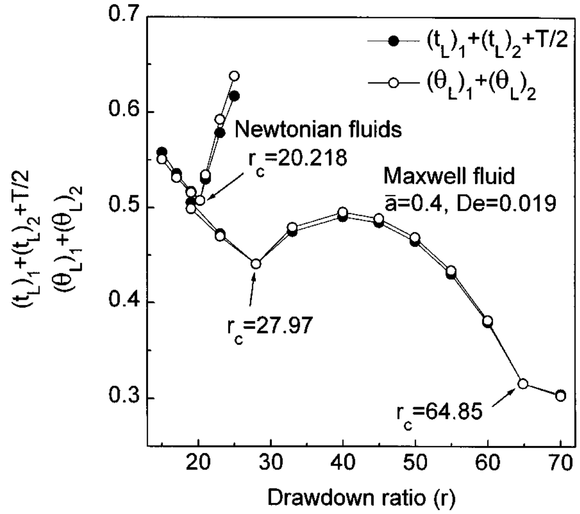

Figure 3. Required traveling time for two successive unity-throughput waves, and allowed traveling time by A_{max} and A_{min} waves plotted against drawdown ratio *r*.

exactly with those by linear stability analysis. Figure 4 shows the contours of the constant throughputs in the *x-t* diagram where the trajectories of unity-throughput waves and A_{max} and A_{min} waves are also shown. The same interrelations among the traveling times in Figure 1 are clearly shown here, also. ($t_L = 0.1220$, $\theta_L = 0.2204$, $T = 0.3936$, $\Delta t = 0.0053$, and $\delta = 0.0183$). To better describe the dynamical character of draw resonance, Figure 5 illustrates the trajectories of draw resonance oscillations in the phase plane exhibiting the limit cycle-like patterns for $r \geq r_c$. When draw resonance is in full swing, such as, $r = 40 > r_c$, the trajectories converge to a teardrop-shaped curve with the point near the origin not being a cusp, and show that draw resonance instability is a supercritical Hopf bifurcation.

Differences between Newtonian and Viscoelastic Fluids Spinning

Finally, an explanation about the differences between Newtonian and viscoelastic spinnings follows. As mentioned before, in the viscoelastic case there is a time delay δ for the spinline tension F to occur after the corresponding area A waves appear at node 1 contrasting with the Newtonian case where F and A waves at node 1 always occur simultaneously. As noted before, this time delay does not, however, affect the criterion for draw resonance because its contributions are cancelled out from the equation. This time delay δ turns out to be in the range of one Deborah number, that is, $\delta = 0.0183$, $De = 0.019$ at the conditions of Figure 1. The reason for the occurrence of this time delay can be explained examining the stress equation of the system, that is, constitutive equation of Eq. 3.

First, Eq. 3 can be rewritten as

$$\tau + \beta(D\tau/Dt) = \dot{\epsilon}\beta/De \tag{10}$$

where

$$\beta = De/\left(1 + De\dot{\epsilon}\left(\bar{a}\sqrt{3} - 2\right)\right),$$
$$(D\tau/Dt) = (\partial\tau/\partial t) + V(\partial\tau/\partial x), \qquad \dot{\epsilon} = \partial V/\partial x \tag{11}$$

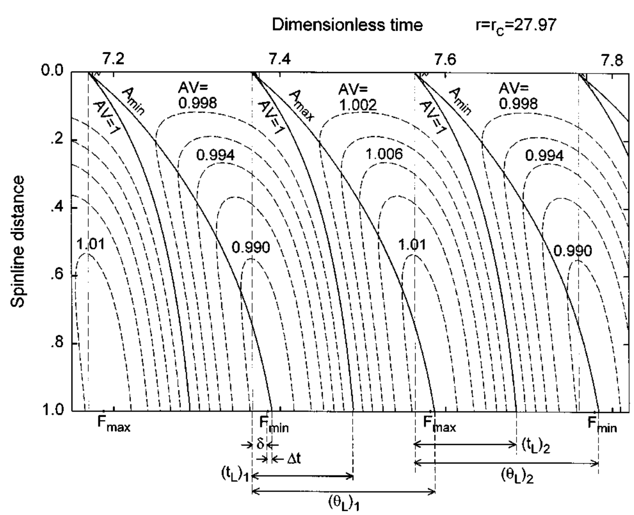

Figure 4. Contours of constant-*AV* curves when *r* = r_c = 27.97 for the case of $\bar{a}$ = 0.4, *De* = 0.019.

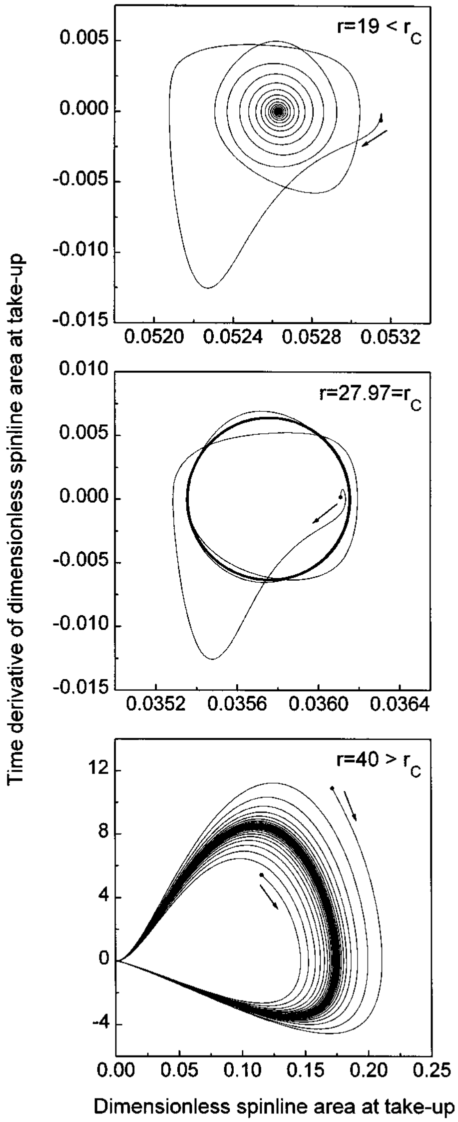

Figure 5. Trajectories of the spinline area at takeup in the phase plane.

Next, substituting $\tau = F/A$ and Eq. 1 into Eq. 10, we get

$$F + \gamma(DF/Dt) = A\dot{\epsilon}\gamma/De \tag{12}$$

where $\gamma = \beta/(1+\beta\dot{\epsilon}) = De/(1+De\dot{\epsilon}(\bar{a}\sqrt{3}-1))$

Equation 12 tells that spinline tension F is described by first-order equation with time constant γ. This γ turns out to be a positive number at node 1, that is, $\gamma \approx 0.01944$, and also is quite close to both De and δ. From this finding, we can say that the response of spinline tension F is delayed according to Eq. 12, explaining why F is delayed behind A waves at node 1, as revealed in Figures 1 and 4.

This time delay by spinline tension has an important implication on the dynamics of viscoelastic spinning, because it delays the system response to disturbances in contrast to the Newtonian cases showing the instantaneous response. This delayed response of the tension means that the tension is made sluggish and less sensitive by viscoelasticity. As Jung et al. (1999b) explained in their analysis of the stabilizing effect of spinline cooling, the spinline tension sensitivity to disturbances is the key to determining the system stability, that is, the more sensitive the tension, the more unstable the system. Hence, this reduction in the tension sensitivity by viscoelasticity stabilizes the spinning as evidenced by larger values for the critical drawdown ratio and smaller values for the severity of draw resonance for viscoelastic cases than those for Newtonian cases. For example, for the case of Figure 1, r_c is 27.97 (20.218 for Newtonian case) and the severity of draw resonance (the ratio of the area peak and trough) is 77.95 at $r = 40$ (43% beyond r_c) in contrast to the Newtonian case having 286.91 at $r = 28.91$ (same 43% beyond r_c). The period of the oscillation of the viscoelastic case becomes smaller than that of the Newtonian case. The details of this viscoelasticity effect on the spinning dynamics will be presented elsewhere (Jung et al., 2000). The effect of any particular process conditions on the stability thus boils down to how much the tension sensitivity is influenced by these process conditions. This stability is also closely connected to productivity and product quality improvement of many industrially important processes.

Conclusions

Following the same method for Newtonian fluids spinning developed by Kim et al. (1996), the isothermal spinning of viscoelastic fluids (an upper convected Maxwell model) has been analyzed to derive the criterion for draw resonance agreeing with the results by the linear stability analysis. This criterion explains the mechanism of draw resonance as to how and why the oscillation starts and persists, as the drawdown ratio is increased beyond its critical value. Differences between Newtonian and viscoelastic fluids spinning are also discussed to reveal that viscoelasticity delays the tension response of the system, rendering the tension less sensitive to make the system more stable. More detailed results will be presented elsewhere (Jung et al., 2000).

Acknowledgments

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