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Analysis of the stabilizing effect of spinline cooling in melt spinning ☆

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Abstract

Despite the fact that understanding of draw resonance in spinning process has steadily advanced with its onset readily predictable by the linear stability analysis method, as [C.J.S. Petrie, Progress Trends Rheol. II (1988) 9] eloquently elaborated, there are still many issues to be answered. For one, the stabilizing effect of spinline cooling has been proven by both experiments and the linear stability analysis but the question of why the cooling performs such a stabilizing role is not yet explained. The same can be said of other process conditions and material properties like elasticity over their roles in spinning stability. The governing physics and the hyperbolic nature of the spinning equations tell us that spinline tension represents the key link in relaying disturbances from the take-up to the spinneret to perpetuate draw resonance. In this simulation study the spinline tension sensitivity to disturbances has been found decreasing as the spinline cooling increases, i.e., stability enhanced by the cooling. This finding explains the success of an ingenious device called draw resonance eliminator of [P.J. Lucchesi, E.H. Roberts, S.J. Kurtz, Plast. Eng. 41 (1985) 87] which renders the spinline tension very insensitive to disturbances using maximum cooling air blown onto the spinline (the film in their case). It also explains why spinning with constant force boundary conditions is always stable by providing the reason that the transmission links between disturbances and the tension are completely disconnected in this case. Newtonian and upper convected Maxwell fluids have been tested to reveal that spinline cooling reduces the tension sensitivity to disturbances, resulting in increased stability. ©1999 Elsevier Science B.V. All rights reserved.

Keywords: Disturbances; Draw resonance; Maxwell fluids; Newtonian fluids; Sensitivity; Spinline tension; Stability; Transmission links

1. Introduction

Thanks to many theoretical and experimental research efforts over the past four decades, understanding of the instability phenomenon occurring in industrially important melt spinning process, i.e. draw

☆ Dedicated to Professor David V. Boger on the occasion of his 60th birthday

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resonance, has continually advanced. The onset of draw resonance is thus readily predictable by linear stability analysis method in terms of the critical values of the drawdown ratio as far as reasonably accurate system model equations are available. However, as Petrie [1] and Larson [3] eloquently elaborated and explained in detail, there are still many issues yet to be addressed and answered regarding the spinning stability.

For example, although the stabilizing effect of spinline cooling has been proven by both experiments and the linear stability analysis, the question of why the cooling performs the stabilizing role is not yet explained. As a matter of fact, this particular issue concerning the spinline cooling goes back to the early stages of draw resonance research. The question of whether the spinline cooling stabilizes or destabilizes melt spinning process was one of the major interests among the early researchers studying the dynamics of melt spinning from both theoretical and experimental viewpoints. There were conflicting reports and some confusion existed in the early literature as to the effect of cooling. The question was finally settled by Fisher and Denn [4] who through stability analysis and experiments convincingly demonstrated that the spinline cooling does stabilize the melt spinning. The critical drawdown ratio at the onset of draw resonance exhibits the smallest value under isothermal spinning conditions, but increases as the extent of cooling increases. The question of why the cooling stabilizes, however, remains unanswered.

There can be many different disturbances to melt spinning process which can trigger instability to set in. Any changes in material properties of fluids like viscosity, elasticity, thermal capacity, density, etc. represent one group of disturbances while changes in process conditions of spinning like spinline velocity, spinline temperature, spinning distance, cooling air velocity, temperature, etc. pose another group of disturbances. The issue of the stability in spinline caused by these various disturbances is industrially very important because it always is closely connected with the subjects of productivity, product quality and profitability of the related industries. So it is in everybody's interest to constantly find ways to improve the stability of the spinning process whatever the final product of the process might be, i.e., fiber, film or sheet. In view of this fact, it is no wonder that during the past several decades the stability of melt spinning process has been studied by many researchers [1–10] and thanks to these extensive studies, the stability issue is fairly settled now as far as the linear stability analysis yielding unequivocal critical values for the onset point, but not yet as far as the fundamental reasons behind the occurrence of draw resonance.

Since Petrie and Denn [10] in 1976 examined the whole gamut of instabilities in polymer processing, many people followed with reports on various aspects of flow instabilities involved in polymer processing operations. Then in 1992 Larson [3] provided another extensive and chronological review dealing with the same subject. Interests in extensional flows in polymer processing also kept pace with these developments [11–17]. Kase and Araki [11], using the frequency response method with a Newtonian fluids model, demonstrated that linear transfer functions between disturbances and process variables convey important necessary information about the responsiveness of the system to disturbances. Liu and Beris [12], employing various numerical schemes, analyzed the stability of the spinning of an upper convected Maxwell model and produced detailed stability diagrams. Devereux and Denn [13] conducted a comparison study of theoretical and experimental results of the frequency response of polymer melt spinning for three different viscoelastic fluid models.

In the present study, we try to explain the fundamental mechanism of why the spinline cooling stabilizes melt spinning by looking into the causality relationship between the disturbances and the process variables utilizing our recent research results [15–17]. Specifically, we delve into the role the spinline tension plays in relaying the effect of disturbances at the take-up to the spinline variables at the spinneret, most notably to the spinline cross-sectional area and thus in forcing waves to travel from the spinneret to the take-up.

How the sensitivity of the spinline tension to disturbances changes with the spinline cooling constitutes the main point of this study.

2. Problem formulation

First we deal with the nonisothermal melt spinning of Newtonian fluids whose governing equations are shown below [5,11].

Continuity equation:

$$\frac{\partial A}{\partial t'} + \frac{\partial (AV)}{\partial z} = 0 \tag{1}$$

Equation of motion:

$$\frac{\partial}{\partial z}(A\sigma) = 0 \tag{2}$$

Constitutive equation:

$$\sigma = \mu_{\mathrm{E}} \frac{\partial V}{\partial z}, \quad \mu_{\mathrm{E}} = \mu_0 \exp\left(\frac{E}{RT_0}\left(\frac{T_0}{T} - 1\right)\right). \tag{3}$$

Energy equation:

$$\frac{\partial T}{\partial t'} + V\frac{\partial T}{\partial z} = -\frac{2\sqrt{\pi}h}{\rho C_p \sqrt{A}}(T - T_{\mathrm{a}}), \quad h = 0.473 \times 10^{-4}\left(\frac{V}{A}\right)^{1/3}\left(1 + \left(8\frac{V_y}{V}\right)^2\right)^{1/6}, \tag{4}$$

These equations are subject to the following boundary conditions.

$$A = A_0, \quad V = V_0, \quad T = T_0 \quad \text{at } z = 0 \text{ for all } t', \tag{5}$$

$$V = V_L = rV_0 \quad \text{at } z = L \text{ for all } t', \tag{6}$$

where A represent spinline cross-sectional area, V is the spinline velocity, T represent the spinline temperature, σ is the spinline axial stress, z is the distance coordinate from the spinneret, t' represent the time, μ_{E} is the extensional viscosity, h is the heat transfer coefficient between the spinline and cooling air, T_{a} represent cooling air temperature, V_y is the cooling air velocity, E represent material activation energy, R is the gas constant. Subscripts 0, L denote spinneret and take-up conditions, respectively.

At time $t = 0^+$, disturbances are introduced to the system with all other conditions maintained the same.

In the above equations, we have adopted the following assumptions. (1) The variations of variables across the spinline cross-section are neglected to result in an one-dimensional model for the system. (2) The origin of the spinning distance coordinate is chosen at the die (extrudate) swell position ignoring the pre-spinneret conditions on the spinline. (3) No crystallization occurs inside the spinline so that the cooling of the spinline is done by the forced convection of cooling air without being interfered with any crystallization heat. (4) All the secondary forces on the spinline, i.e., gravity, air drag, surface tension and inertia, are neglected. The last assumption was adopted here to clearly expose the cooling effect in the problem formulation without effects from other secondary forces blurring the point. The inclusion of secondary forces into the model, though, would not change the fundamental aspects of the study.

The above Eqs. (1)–(6) are also rendered dimensionless as follows.

Dimensionless continuity equation:

$$\frac{\partial a}{\partial t}+\frac{\partial (av)}{\partial x}=0. \tag{7}$$

Dimensionless equation of motion with constitutive equation included:

$$\frac{\partial}{\partial x}\left(a\,\exp\left(k\left(\frac{1}{\theta}-1\right)\right)\frac{\partial v}{\partial x}\right). \tag{8}$$

Dimensionless equation of energy:

$$\frac{\partial \theta}{\partial t}+v\frac{\partial \theta}{\partial x}=-\mathrm{St}\,v^{1/3}a^{-5/6}(\theta-\theta_{\mathrm{a}})\left(1+\left(8\frac{v_y}{v}\right)^2\right)^{1/6},\quad \mathrm{St}=\frac{1.67\times 10^{-4}L}{\rho C_p A_0^{5/6}V_0^{2/3}}. \tag{9}$$

Boundary conditions:

$$a=1,\quad v=1,\quad \theta=1\quad \text{at } x=0 \text{ for all } t, \tag{10}$$

$$v=r\quad \text{at } x=1 \text{ for all } t. \tag{11}$$

where the dimensionless variables and parameters denote

$$t=\frac{t'V_0}{L},\quad x=\frac{z}{L},\quad a=\frac{A}{A_0},\quad v=\frac{V}{V_0},\quad v_y=\frac{V_y}{V_0},\quad \theta=\frac{T}{T_0},\quad \theta_{\mathrm{a}}=\frac{T_{\mathrm{a}}}{T_0},\quad k=\frac{E}{RT_0} \tag{12}$$

The governing equations for melt spinning of upper convected Maxwell fluids are the same as in the above except for the constitutive equation which is shown below [4,9,12,14]

$$\tau+\mathrm{De}\left(\frac{\partial \tau}{\partial t}+v\frac{\partial \tau}{\partial x}-2\tau\frac{\partial v}{\partial x}\right)=2\frac{\mathrm{De}}{\mathrm{De}_0}\frac{\partial v}{\partial x},\quad \mathrm{De}=\mathrm{De}_0\exp\left(k\left(\frac{1}{\theta}-1\right)\right), \tag{13}$$

where, $\tau=\sigma L/\mu_0 V_0$, $\mathrm{De}_0=\lambda_0 V_0/L$. An additional dimensionless number, Deborah number (De_0), here characterizes the dynamic behavior of this viscoelastic fluid in melt spinning. The temperature dependency of this Deborah number in Eq. (13) is through the viscosity function like the one shown in Eq. (3), and not through the modulus which is generally considered temperature independent in Maxwell models. The stress equation in Eq. (13) here is an approximation in that it represents the axial stress while the radial stress could be obtained following an approximation given by Beris and Liu [14]. The procedure to conduct simulation is the same as for the Newtonian fluids.

3. Results and discussions

Now we are in a position to investigate, using simulation results, the transient behavior of the system as to how and why the spinline cooling stabilizes melt spinning. As Table 1 shows, with the increasing spinline cooling (the increasing cooling air velocity), the critical drawdown ratio (i.e., ratio of the take-up velocity to the spinline velocity at the spinneret) for both Newtonian fluids and a Maxwell fluid at the onset of draw resonance increases, i.e., stability is improved. The Deborah number of 0.001 has been

Table 1
Critical drawdown ratio for both Newtonian fluids and a Maxwell fluid in nonisothermal spinning (St = 0.1, k = 5.8, θ_a = 0.543)

Cooling conditions		r_c
(a) Newtonian fluids		
Isothermal spinning		20.218
Nonisothermal spinning	$v_y = 0$	30.50
	$v_y = 7.7$	52.50
	$v_y = 10$	60.05
	$v_y = 20$	93.32
	$v_y = 30$	135.18
Cooling conditions		r_c
(b) A Maxwell fluid ($De_0 = 0.001$)		
Isothermal spinning		20.93
Nonisothermal spinning	$v_y = 0$	34.78
	$v_y = 3.07$	52.50
	$v_y = 5$	77.30
	$v_y > 10$	Always stable

chosen here to clearly show the draw resonance because higher numbers would make this upper convected Maxwell system stable. This stabilizing effect of spinline cooling is shown to be more pronounced in the case of Maxwell fluid than in Newtonian fluids. The results here have been obtained by the linear stability analysis but the same can be produced by the transient simulation of the system.

In order to clearly demonstrate the stabilizing effect of the spinline cooling, we have performed simulation runs at different cooling air velocities. Here we have introduced a step disturbance to the take-up velocity, i.e., a 5% increase in v_L, at time 0^+. The drawdown ratio was thus 50 before the introduction of the disturbance and then 52.5 after a new take-up velocity is imposed.

Figs. 1(a) and (b) show the transient behavior of the spinline cross-sectional area at the take-up for Newtonian fluids and a Maxwell fluid, respectively, when different cooling air velocities are applied. In agreement with Table 1, the three cases each in Figs. 1(a) and (b) exhibit the system in instability, at the onset of instability, and in stability, respectively, clearly showing that the cooling stabilizes the system. Although not shown in Fig. 1, as time elapses sufficiently long, the cases of instability and onset of instability in Figs. 1(a) and (b) exhibit steady oscillations of draw resonance, whereas the cases of stability exhibit oscillations dying out.

Now we set out to analyze why this increased cooling results in stabilizing the system by looking into the key link which perpetuates the draw resonance instability, i.e., the spinline tension. As can be found in Hyun [8,9], Kase and Araki [11], and Kim et al. [15], the reason for this is explained as follows. Due to the hyperbolic nature of the melt spinning equations [8,9,14,15,16], any disturbances to the melt spinning system, whether they are in material properties or in process conditions, travel the spinline in the form of many waves including the cross-sectional area wave from the spinneret to the take-up and then go out of the system by passing through the take-up. When these disturbances pass through the take-up position, they are bound to cause the existing spinline tension to change because the tension being the product of the stress and the cross-sectional area is determined by the conditions at the take-up. This changed spinline tension then immediately causes a new cross-sectional area disturbance wave (with an opposite sign compared to the previous disturbance) to appear at the spinneret, which again travels toward the

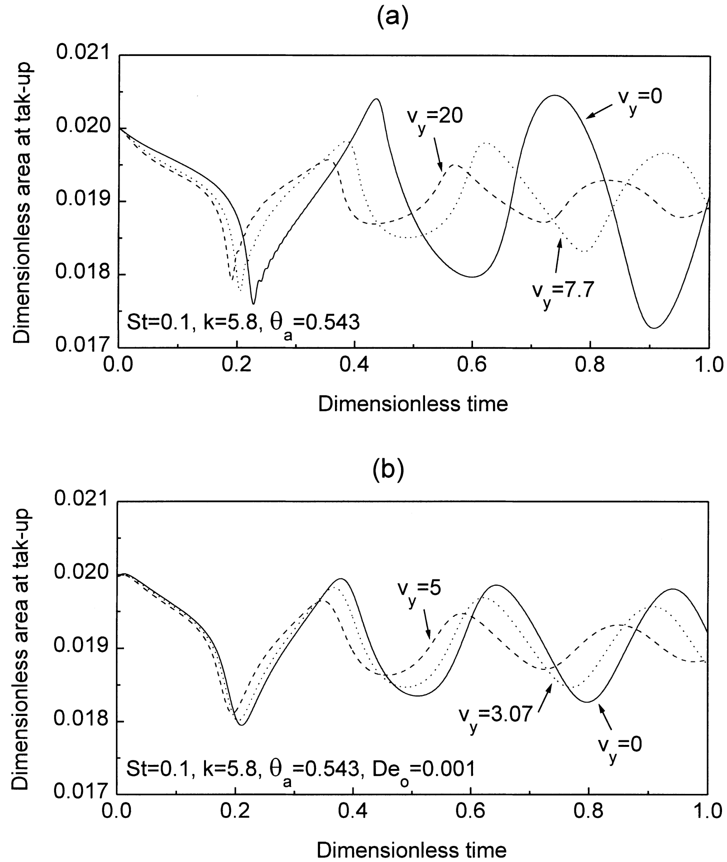

Fig. 1. Transient behavior of the spinline cross-sectional area at take-up for (a) Newtonian fluids and (b) a Maxwell fluid, when a disturbance in take-up velocity is introduced.

take-up. When this new disturbance wave arrives and passes through the take-up, the spinline tension is changed again but this time with the opposite sign of the previous change. Another cross-sectional area wave with the opposite sign then appears at the spinneret and travels, and the whole cycle repeats itself.

As explained above, the perpetuation of the draw resonance instability is possible as the spinline tension plays the key role in relaying the changed conditions at the take-up to the spinneret in an instantaneous fashion. So the spinning instability depends on how sensitive this spinline tension is to the changed conditions at the take-up. The more sensitive the tension is to the changes at the take-up which are of course caused by the disturbances to the system, the less stable the spinning system becomes. On the other hand, if for some reasons the spinline tension is not sensitive at all, i.e., totally insensitive to changes or disturbances of the system, then the perpetual transmission mechanism of draw resonance breaks down and the system becomes absolutely stable all the time. This is the case when we maintain constant force boundary conditions for the spinning system. Ordinarily, however, with constant take-up velocity boundary condition the spinline tension is not constant but rather sensitive to the changing conditions of the system, i.e., disturbances, and so draw resonance instability persists as long as the drawdown ratio of spinning exceeds its critical value.

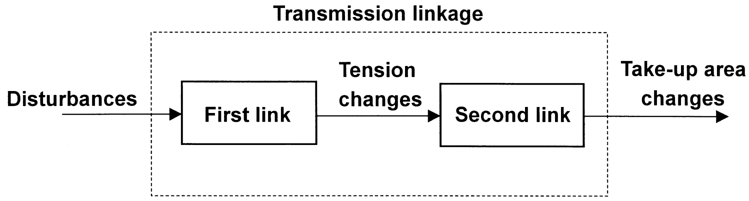

Fig. 2. Schematic diagram illustrating the transmission linkage between disturbances and the spinline cross-sectional area at take-up.

Table 2
Sensitivities of spinline tension and spinline cross-sectional area at take-up to a disturbance in the take-up velocity ($\mathrm{St}=0.1$, $k=5.8$, $\theta_a=0.543$, $r=50\rightarrow 52.5$)

v_y		F (spinline tension)	$\lvert\Delta \ln F\rvert$ (first sensitivity in Fig. 2)	$\lvert\Delta \ln A_L\rvert/\lvert\Delta \ln F\rvert$ (second sensitivity in Fig. 2)	$\lvert\Delta \ln A_L\rvert$ (overall sensitivity in Fig. 2)
(a) Newtonian fluids					
0	Unstable	4.6071	0.08147	1.5742	0.1283
7.7	Critical	5.7269	0.06931	1.7037	0.1181
10	Stable	5.9222	0.06710	1.7255	0.1158
20	Stable	6.4928	0.06152	1.7841	0.1098
(b) A Maxwell fluid ($\mathrm{De}_0=0.001$)					
0	Unstable	4.8975	0.07572	1.5170	0.1149
3.07	Critical	5.6064	0.06912	1.5800	0.1092
5	Stable	5.8753	0.06553	1.6081	0.1054
10	Stable	6.3687	0.05972	1.6597	0.0991
20	Stable	7.0254	0.05341	1.7264	0.0922

In explaining the above point more clearly, a schematic diagram like Fig. 2 is helpful where the transmission linkage between disturbances and the final cross-sectional area at take-up through the intermediate spinline tension is illustrated. There are two links connecting disturbances and spinline area, i.e., the first link determines the tension sensitivity to disturbances while the second does the area sensitivity to tension. The fact that the spinline tension is decided by the conditions at the take-up and the spinline dynamics is governed by this tension tells us that the transmission links are necessarily in a series type as shown in Fig. 2 [11]. The overall sensitivity of the area to disturbances is then the product of the two intermediate sensitivity values.

Among these two sensitivities, the first one, i.e., the tension sensitivity, is more important. This is because, as explained before, of the key role played by the spinline tension in perpetuating sustained oscillation of spinline variables. The important point is then how sensitive to disturbances the spinline tension is. The less sensitive it is, the more stable the system becomes. So in the present study we look into this sensitivity question by computing the tension changes with respect to the various disturbances introduced to the system.

For the cases of different cooling conditions including those of Fig. 1, we thus have computed the tension sensitivity. This is obtained in the form of the absolute values of logarithmic changes of tension between the peak and trough of the transient tension curve which was produced introducing the disturbance of a 5% increase in the take-up velocity. Table 2 shows the results for Newtonian fluids and a Maxwell fluid, respectively.

Table 3
Sensitivities of spinline tension and spinline cross-sectional area at take-up to a disturbance in the extrusion temperature (St = 0.1, $k = 5.8$, $\theta_a = 0.543$, $\theta(0) = 1 \to 0.95$)

v_y		F (spinline tension)	$\lvert\Delta \ln F\rvert$ (first sensitivity in Fig. 2)	$\lvert\Delta \ln A_L\rvert / \lvert\Delta \ln F\rvert$ (second sensitivity in Fig. 2)	$\lvert\Delta \ln A_L\rvert$ (overall sensitivity in Fig. 2)
(a) Newtonian fluids					
0	Unstable	4.6071	1.2269	2.6204	3.2150
5	Unstable	5.4883	0.7074	3.7013	2.6183
10	Stable	5.9222	0.5419	4.5017	2.4395
20	Stable	6.4928	0.3833	5.8784	2.2532
(b) A Maxwell fluid ($De_0 = 0.001$)					
0	Unstable	4.8975	0.6397	2.5313	1.6193
5	Stable	5.8753	0.3500	3.8023	1.3308
10	Stable	6.3687	0.2530	4.8225	1.2201
20	Stable	7.0254	0.1627	6.7843	1.1038

As the extent of the spinline cooling increases, the tension sensitivity indeed decreases, indicating a decreasing trend of the first link in Fig. 2 between disturbances and the tension. As for the second sensitivity, Table 2 shows that its value actually increases with increasing cooling. But its extent of increase is smaller than the extent of decrease in the first tension sensitivity so that the overall effect of the disturbance on the spinline area decreases with increasing cooling, as shown in the last columns of Table 2. The reason why increased spinline cooling stabilizes the system is thus that it reduces the tension sensitivity to disturbances.

Here one comment might be added on why we have computed the values of the tension sensitivity here and not the initial growth rate of disturbances commonly mentioned in stability studies [4]. The reason is rather simple. While in linear stability analysis the sign and the magnitude of the real part of the largest eigenvalues determines the growth rate of the most dominant dynamic mode, the physics behind this dynamics is frequently lost when numerical discretization is introduced. In order to expose the role played by the spinline tension in deciding the stability of the spinning process, we thus computed their sensitivity values to disturbances. The information contained in eigenvalues and sensitivities might be the same, but the physical meaning is clear in the latter while not in the former.

Table 3 exhibits another example with a different disturbance, i.e., a step change in the extrusion temperature for Newtonian fluids and a Maxwell fluid, respectively. As in Table 2, increased spinline cooling does stabilize the spinning system because it again reduces the tension sensitivity in the first link of Fig. 2 while the sensitivity in the second link moves in the opposite direction as compared to the first link but to a lesser extent. The overall stabilizing effect of the spinline cooling is thus always decided by the sensitivity of the first link, i.e., the tension sensitivity.

Finally we consider the question of why the increased cooling then reduces the tension sensitivity as observed in Tables 2 and 3. Partial answer to this question lies in the ingenious invention by Lucchesi et al. [2] called the draw resonance eliminator that Petrie [1] also recognized as a significant example where practical people are ahead of the theorists in contributing toward better understanding of draw resonance. In this device, a maximum cooling air is blown onto the spinline (in their case onto the cast film) so that the tension is maintained on a highest possible level and then the tension is very insensitive to any disturbances of the system. Here we can say that if the spinline tension is held on its maximum limit, its sensitivity to disturbances tends to be minimum. In other words, the tension sensitivity decreases with increasing spinline tension. This is exactly what happens when the spinline cooling increases, as Tables 2 and 3 corroborate the point.

Although we have shown just two disturbances in this study, i.e., take-up velocity and extrusion temperature, there are many disturbances possible as mentioned in the introduction of this paper. A number of different disturbances were tested in this study but the results were all similar and so they are not produced here. In addition to the step disturbances, impulse disturbances have also been tried in this sensitivity study to yield the same results. The stabilizing effect of the spinline cooling has been found unchanged regardless of the kinds of disturbances.

As mentioned in Section 1, the effects of other process conditions on the spinline stability can also be analyzed equally well following the same method as here. For example, the question about the effect of secondary forces of gravity, air drag and inertia, or that of fluid viscoelasticity on the spinline stability which is traditionally solved using a linear stability analysis, could be better answered using the present analysis of the sensitivity of tension. We can thus provide a fundamental reasoning for the stability which the linear stability calculations alone normally cannot explain. The basic mechanism of the spinning instability known as draw resonance therefore, depends upon how well the spinline tension relays the effect of disturbances at the take-up to the cross-sectional area. The effect of any particular process conditions on the stability is then dependent on how much the tension sensitivity is influenced by these process conditions.

4. Conclusions

The experimentally and theoretically-observed fact that melt spinning is stabilized by increasing spinline cooling has been investigated to reveal the fundamental reasons behind. Both Newtonian and Maxwell fluids models have been used in simulating nonisothermal melt spinning processes with different levels of spinline cooling. The results indicate the reason for the stabilizing effect of the spinline cooling to be that the sensitivity of spinline tension to disturbances decreases with increased cooling. Since the less the spinline tension is affected by disturbances, the more the spinning becomes stable, the decreased tension sensitivity is the key for explaining why increased cooling always stabilizes spinning. This interpretation is in line with the basic idea behind the ingenious device called draw resonance eliminator developed by Lucchesi et al. [2] which guarantees stability by blowing maximum cooling air onto the spinline. The high level of the spinline tension resulted by the increased cooling always minimizes the tension sensitivity to disturbances which dominates the overall transmission linkage between disturbances to the final spinline area at the take-up.

This stabilizing effect of spinline cooling always exists irrespective of kinds of disturbances to spinning, i.e., regardless whether disturbance is a step change or an impulse, and whether disturbance is a material property change or process changes. Other subjects like the effect of viscoelasticity of fluids and inertia on the spinning stability can also be studied employing the same methodology of this study.

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