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Effect of fluid viscoelasticity on the draw resonance dynamics of melt spinning

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Abstract

The effect of fluid viscoelasticity on the draw resonance dynamics of melt spinning has been examined using White-Metzner and Phan Thien-Tanner fluid models into the governing equations of the process, in a continued effort to study the effects of process conditions and material properties on draw resonance, following up the earlier study [J. Non-Newtonian Fluid Mech. 87 (1999) 165] dealing with the effect of spinline cooling on the same draw resonance. Whether or not the fluid viscoelasticity stabilizes melt spinning has turned out to coincide with whether or not the spinline tension sensitivity decreases with the increasing fluid viscoelasticity. This is because the spinning stability is always enhanced by a decrease in tension sensitivity to process disturbances and this tension sensitivity was then found in the said earlier study to be moving opposite to the level of the spinline tension: the higher spinline tension, the smaller tension sensitivity. It has been found in the present study that the effect of fluid viscoelasticity on spinning stability can be classified into two diametrically different kinds: for extension-thickening fluids an increase in viscoelasticity increases tension, decreases tension sensitivity and, thus, stabilizes the spinning, whereas it decreases tension, increases tension sensitivity and, thus, destabilizes the spinning of extension-thinning fluids.

Keywords: Draw resonance; Melt spinning; Phan Thien-Tanner fluids; Spinline tension sensitivity; Viscoelasticity; White-Metzner fluids

1. Introduction

Draw resonance, one of major instabilities in polymer processing, arises as the drawdown ratio is increased beyond its critical value and is manifested by sustained periodic variations in spinline variables such as cross-sectional area and tension. Ever since Christensen [1] and Miller [2] first discovered the phenomenon and aptly named it as such, draw resonance has been the subject of research for many

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people around the world since it is closely related to the industrially important productivity issue and the academically interesting stability topic as well.

Thanks to those research efforts in the past four decades, draw resonance is now fairly well understood. (see, [3–7]) First, draw resonance, a supercritical Hopf bifurcation, is a hydrodynamic instability, not a viscoelastic one, albeit altered by fluid viscoelasticity and, thus, even Newtonian fluids can exhibit it. Second, its onset is readily and accurately predicted employing linear stability analysis if the governing equations are provided. However, the physics behind this draw resonance as to why and how it occurs has not been fully understood.

In an effort to shed light further on this instability, a new concept was tried focusing on the hyperbolic nature of the system, i.e. kinematic waves traveling along the spinline [8–12]. Specifically, comparing the traveling times of throughput waves and those of maximum and minimum spinline cross-sectional area waves, a new criterion for draw resonance was derived. As the draw down ratio increases, this stability criterion determines its critical value at the onset point exactly agreeing with the results by the linear stability analysis.

The next important issue surrounding the draw resonance is what kinds of effects process conditions and material properties have on spinning stability and why they have such effects. Among them the most notable are spinline cooling, inertia (spinning velocity) and fluid viscoelasticity. In this study we deal with the fluid viscoelasticity as to how and why it has stabilizing or destabilizing effect on spinning.

2. Description of the system

The isothermal melt spinning of upper convected Maxwell (White-Metzner) fluids and Phan Thien-Tanner fluids have the following dimensionless equations.

Continuity equation:

$$\frac{\partial a}{\partial t} + \frac{\partial (av)}{\partial x} = 0, \tag{1}$$

where

$$a = \frac{A}{A_0}, \quad v = \frac{V}{V_0}, \quad t = \frac{t' V_0}{L}, \quad x = \frac{z}{L}.$$

Equation of motion:

$$\frac{\partial (a\tau)}{\partial x} = 0, \quad \text{where } \tau = \frac{\sigma L}{2\eta_0 V_0}. \tag{2}$$

Constitutive equation:

1. White-Metzner model:

$$\tau \left(1 + \bar{a}\sqrt{3} De \frac{\partial v}{\partial x}\right) + De\left(\frac{\partial \tau}{\partial t} + v\frac{\partial \tau}{\partial x} - 2\tau\frac{\partial v}{\partial x}\right) = \frac{\partial v}{\partial x}, \tag{3}$$

where

$$\lambda = \frac{\lambda_0}{1 + \bar{a}\sqrt{3} De(\partial v/\partial x)}, \quad De = \frac{\lambda_0 V_0}{L}.$$

2. Phan Thien-Tanner model:

$$\exp(2\varepsilon De\tau)\tau + De\left(\frac{\partial\tau}{\partial t} + v\frac{\partial\tau}{\partial x} - 2(1-\xi)\tau\frac{\partial v}{\partial x}\right) = \frac{\partial v}{\partial x}. \quad (4)$$

These dimensionless equations are subject to the following boundary conditions.

$$t = 0: \quad a = a_s,\ V = V_s,\ \tau = \tau_s \quad \text{for}\ 0 < x < 1,$$

$$t > 0: \quad a = a_0 = 1,\ v = v_0 = 1 \quad \text{at}\ x = 0,$$

$$v = v_L = r(1+\epsilon) \quad \text{at}\ x = 1, \quad (5)$$

where A is the spinline cross-sectional area, a the dimensionless A, V the spinline velocity, v the dimensionless V, σ the spinline axial stress, τ the dimensionless σ, z the distance coordinate from spinneret, x the dimensionless z, t' the time, t the dimensionless t', L the spinning distance from spinneret to the take-up, De the Deborah number, $\bar{a}$, ε, ξ are the parameters, η_0 the zero shear rate viscosity, λ the material relaxation time, $\lambda_0 = \lambda$ at zero extension rate, r the drawdown ratio, ϵ the constant representing the initial disturbance at the take-up, and subscripts 0, L, s denote spinneret, take-up, and steady state conditions, respectively.

Several assumptions have been incorporated. First, the variations of variables across the spinline cross-section are neglected to result in a one-dimensional model. Second, the origin of the distance coordinate is chosen at the die (extrudate) swell position, thus, ignoring the pre-spinneret conditions on the spinline. Third, all the secondary forces, i.e. gravity, air drag, surface tension, and inertia are neglected as shown in Eq. (2). Fourth, in Eqs. (3) and (4) instead of a full set of equations including the radial stress equation only the axial stress equation is solved while the radial stress is obtainable by the method of Beris and Liu [6].

3. Simulation results and discussions

In this study, we analyze the effect of fluid viscoelasticity on spinning stability as imbedded in De (Deborah number) defined in Eqs. (3) and (4). As Fig. 1 shows, the spinning stability for White-Metzner fluids improves with the increasing De for the fluids which have smaller values for $\bar{a}$, i.e. $\bar{a} < 0.57$ whereas the stability worsens for those having larger values, $\bar{a} > 0.57$. Similar findings are shown in Fig. 2 for the spinning of Phan Thien-Tanner fluids, i.e. the stability improves with increasing De for the fluids of $\xi < 0.5$, while it worsens for $\xi > 0.5$ fluids. This kind of dichotomous behavior of viscoelastic fluids in spinning is not new since there have been many corroborating experimental results (e.g. [13,14]) dealing with thermoplastic polymers such as LDPE and HDPE.

The above different results in Figs. 1 and 2 caused by the fluid viscoelasticity depending on the magnitude of the parameters are a direct contrast to those of the spinline cooling case which was found in the earlier study [15] to always have stabilizing effect on spinning regardless of the magnitude of the parameters. In an attempt to shed some light on these interesting findings, the extensional viscosities at different spinline positions have been plotted against the extension rates at the same positions for the above spinning cases. As Fig. 3 shows, the fluids having smaller values for either parameter $\bar{a}$ or ξ exhibit extension-thickening behavior while those having larger values extension-thinning behavior. From the results in Figs. 1–3, it then can be said that for extension-thickening fluids the fluid viscoelasticity has stabilizing effect on spinning whereas it has destabilizing effect for extension-thinning fluids.

Before examining why the fluid viscoelasticity can have these stabilizing or destabilizing effects on spinning, first we consider the mechanism how the spinline stability is affected by the changes in process conditions and/or material properties. As shown in the earlier study [15] about the spinline cooling effect on spinning, the key link perpetuating the draw resonance in spinning is the spinline tension. Due to the hyperbolic nature of the melt spinning equations, any disturbances to the melt spinning system, whether they are in material properties or in process conditions, travel the spinline in the form of many waves including the cross-sectional area wave from the spinneret to the take-up and then go out of the system by passing through the take-up. When these disturbances pass through the take-up position, they are bound to cause the spinline tension to change because the spinline tension is the product of the spinline stress and the cross-sectional area. This changed spinline tension then causes a new cross-sectional area disturbance wave (with an opposite sign compared to the previous disturbance) to appear at the spinneret, which again travels toward the take-up. When this new disturbance wave arrives and passes through the take-up, the spinline tension is changed again but this time with the opposite sign of the previous change. Another cross-sectional area wave with the opposite sign then appears at the spinneret and travels, and the whole cycle repeats itself.

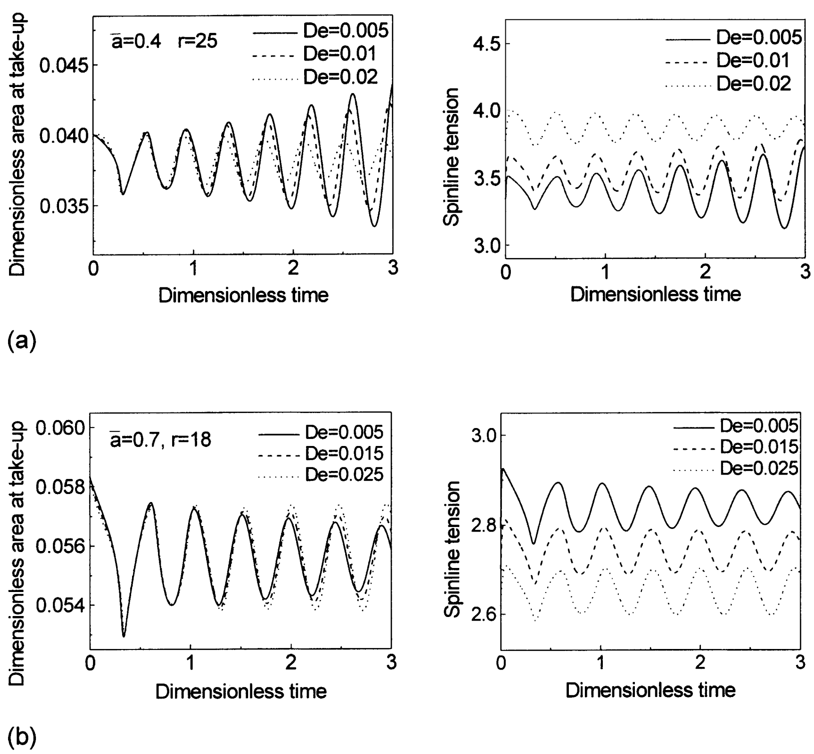

Fig. 1. Transient response of the cross-sectional area at take-up and spinline tension of the spinning of two different White-Metzner fluids: (a) an extension-thickening case, and (b) an extension-thinning case.

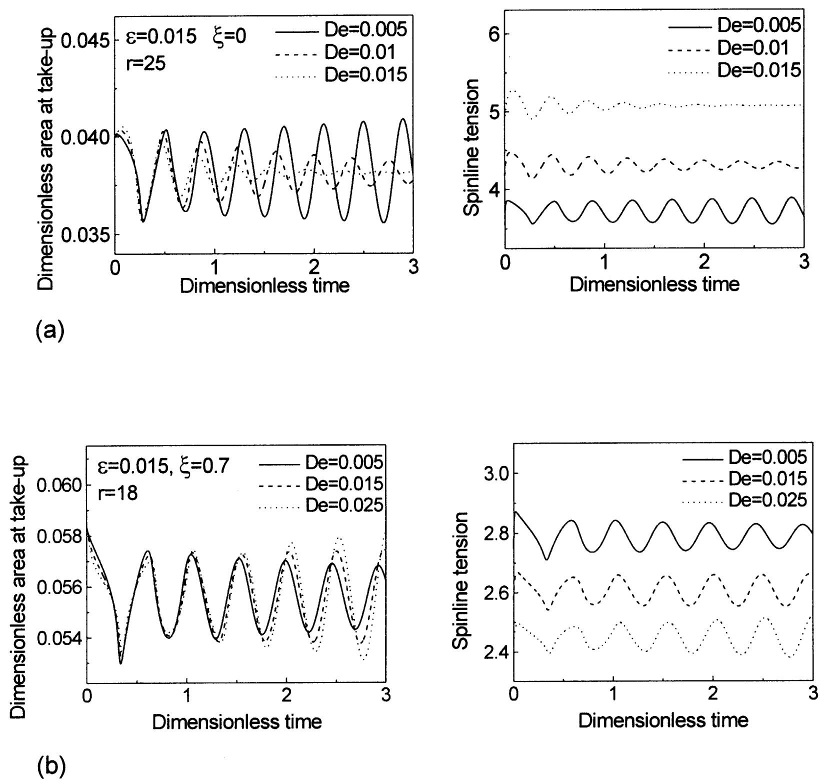

Fig. 2. Transient response of the cross-sectional area at take-up and spinline tension of the spinning of two different Phan Thien-Tanner fluids: (a) an extension-thickening case, and (b) an extension-thinning case.

As explained above, the perpetuation of the draw resonance instability is possible as the spinline tension plays the key role in relaying the changed conditions at the take-up to the spinneret, in an instantaneous fashion for Newtonian fluids and with a small delay in the order of Deborah number for viscoelastic fluids [16]. So the spinning instability depends on how sensitive this spinline tension is to the changed

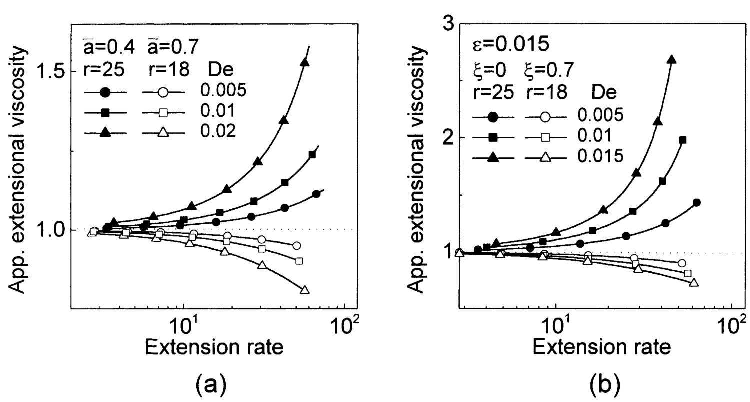

Fig. 3. Spinline extensional behavior of (a) White-Metzner fluids, and (b) Phan Thien-Tanner fluids.

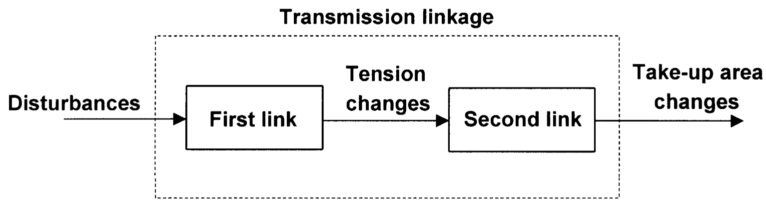

Fig. 4. Schematic diagram illustrating the transmission linkage between disturbances and the spinline cross-sectional area at take-up.

conditions at the take-up. The more sensitive the tension is to the changes at the take-up which are of course caused by the disturbances to the system, the less stable the spinning system becomes. On the other hand, if for some reasons the spinline tension is not sensitive at all, i.e. totally insensitive to changes or disturbances of the system, then the perpetual transmission mechanism of draw resonance breaks down and the system becomes absolutely stable all the time. This is the case when we maintain constant force boundary conditions for the spinning system. Ordinarily, however, with constant take-up velocity boundary conditions the spinline tension is not constant but rather sensitive to the changing conditions of the system, i.e. disturbances, and so draw resonance instability persists as long as the drawdown ratio of spinning exceeds its critical value.

In explaining the above points more clearly, a schematic diagram like Fig. 4 is helpful where the transmission linkage between disturbances and the final cross-sectional area at take-up through the intermediate spinline tension is illustrated. There are two links connecting disturbances and spinline area, i.e. the first link determines the tension sensitivity to disturbances while the second does the area sensitivity to tension. The fact that the spinline tension is decided by the conditions at the take-up and the spinline dynamics is governed by this tension tells us that the transmission links are necessarily in a series type [17] as shown in Fig. 4. The overall sensitivity of the area to disturbances is then the product of the two intermediate sensitivity values.

Among these two sensitivities, the first one, i.e. the tension sensitivity, is more important. This is because, as explained before, of the key role played by the spinline tension in perpetuating sustained oscillation of spinline variables. The important point is then how sensitive to disturbances the spinline tension is. The less sensitive it is, the more stable the system becomes. So in the present study we look into this sensitivity question by computing the tension changes with respect to disturbances.

For the cases of different viscoelasticity values including those of Figs. 1 and 2, we, thus, have computed the tension sensitivity. This is obtained in the form of the absolute values of logarithmic changes of tension between the peak and trough of the transient tension curve which was produced introducing the disturbance of 5% increase in the take-up velocity. Tables 1 and 2 show the results for White-Metzner fluids and Phan Thien-Tanner fluids, respectively.

Both tables show that for extension-thickening fluids, i.e. small values for $\bar{a}$ or ξ, increasing viscoelasticity (increasing *De*) brings about increased tension, decreased tension sensitivity (the first sensitivity) and decreased cross-sectional area fluctuation (severity of draw resonance), whereas for extension-thinning fluids the opposite is true, i.e. increasing viscoelasticity brings about decreased tension, increased tension sensitivity and increased cross-sectional area fluctuation. As for the second sensitivity, the both tables show that its value exhibits the opposite trend to that of the first sensitivity (tension sensitivity) as the viscoelasticity changes. But the extent of changes in the second sensitivity is smaller than that of the

Table 1
Sensitivities of spinline tension and spinline cross-sectional area at take-up to a disturbance in the take-up velocity for White-Metzner fluids spinning

De	Stability	F (spinline tension)	$\lvert\Delta \ln F\rvert$ (first sensitivity = tension sensitivity)	$\lvert\Delta \ln A_L\rvert/\lvert\Delta \ln F\rvert$ (second sensitivity)	$\lvert\Delta \ln A_L\rvert$ (overall sensitivity = a severity of draw resonance)
Extension thickening behavior ($r = 25$, $\bar{a} = 0.4$)					
0.005	Unstable	3.3421	0.09579	1.5357	0.1471
0.01	Unstable	3.4751	0.08524	1.5615	0.1331
0.02	Stable	3.7756	0.05281	1.6664	0.0880
Extension thinning behavior ($r = 18$, $\bar{a} = 0.7$)					
0.005	Stable	2.7937	0.03732	1.5702	0.05860
0.015	Stable	2.7026	0.03835	1.5935	0.06111
0.025	Unstable	2.6172	0.03938	1.6427	0.06469

Table 2
Sensitivities of spinline tension and spinline cross-sectional area at take-up to a disturbance in the take-up velocity for Phan Thien-Tanner fluids spinning

De	Stability	F (spinline tension)	$\lvert\Delta \ln F\rvert$ (first sensitivity = tension sensitivity)	$\lvert\Delta \ln A_L\rvert/\lvert\Delta \ln F\rvert$ (second sensitivity)	$\lvert\Delta \ln A_L\rvert$ (overall sensitivity = a severity of draw resonance)
Extension thickening behavior ($r = 25$, $\varepsilon = 0.015$, $\xi = 0.0$)					
0.005	Unstable	3.6395	0.07802	1.5496	0.1209
0.01	Stable	4.1749	0.03820	1.6073	0.0614
0.015	Stable	4.8718	0.01087	1.7111	0.0168
Extension thinning behavior ($r = 18$, $\varepsilon = 0.015$, $\xi = 0.7$)					
0.005	Stable	2.7466	0.03809	1.5673	0.05970
0.015	Unstable	2.5761	0.03971	1.5857	0.06297
0.025	Unstable	2.4270	0.04001	1.6381	0.06554

first one so that the overall effect of the disturbances on the cross-sectional area, or the severity of draw resonance, is wholly determined by the first tension sensitivity to disturbances.

4. Conclusions

We now draw conclusions from the above findings regarding the effect of fluid viscoelasticity on spinning stability: whether fluid viscoelasticity stabilizes or destabilizes spinning process depends on whether the viscoelasticity increases spinline tension or not. Namely, in the extension-thickening cases where spinline tension is increased by increasing viscoelasticity, spinning is stabilized by viscoelasticity while in the extension-thinning cases the opposite is true. The reason for this is of course, as explained above, that increased or decreased tension corresponds to decreased or increased tension sensitivity, respectively, which then stabilizes or destabilizes the spinning.

The analysis method we have used in this study in explaining how and why any particular process conditions or material properties have certain effects on stability, can also be employed to analyze the effects of process conditions other than the present viscoelasticity, such as spinline cooling or inertia (spinning velocity) on spinning stability. For example, the spinline cooling has always a stabilizing effect regardless of the magnitude of the parameters of $\bar{a}$ or ξ as shown by Jung et al. [15]. The spinning velocity, i.e. inertia, has also a similar stabilizing effect and these results will be reported elsewhere.

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