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Short communication

A direct synthesis tuning method of unstable first-order-plus-time-delay processes

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Abstract

A direct synthesis tuning method is proposed for the PI controller settings of unstable first-order-plus-time-delay processes. Unlike hitherto-known PI setting rules which often result in overshoots in time response or require the modification of the feedback control structure, this method ensures the overdamped response as desired while retaining the conventional PI control structure. This enhanced performance is possible by introducing a first-order set-point filter and applying simple rules for setting the values of the controller parameters without having any tuning parameters. The comparison with both conventional PI controllers and two-stage IMC method reveals that the proposed method produces not only smooth overdamped closed-loop response for set-point changes, but also fast regulatory control response for load changes. These responses are also shown to be quite robust against the uncertainties of the parameters as well as against the noise in the signal. The stability conditions for the processes having a large time delay or different ratios of time delay/time constant have been investigated as well. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Research on tuning methods of the conventional controllers for unstable processes has recently been active. (e.g. [1–5]) Kavdia and Chidambaram [2] studied an unstable bio-reactor process using both the on-line identification method of Yuwana and Seborg [6] and the PI tuning rule by Venkatashankar and Chidambaram [3]. In order to overcome an overshoot problem on the set-point tracking performance, Jacob and Chidambaram [1] successfully developed a new tuning rule incorporating both a two-stage P–PI control structure and the IMC–PID tuning rule [7].

In this paper we seek a new tuning method which produces smooth closed-loop response with strong robustness against the parameters uncertainties but does not require any modified control structure nor tuning parameters. A set-point filter enables us to compensate for the overshoots under the conventional PI control structure, and a simple searching technique provides us with easy calculation of PI parameters. The proposed method is used for regulatory control as well as for set-point tracking.

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2. Closed-loop response

The transfer function of an unstable first-order-plus-time-delay process is given by

$$G_p = \frac{K_P e^{-\theta_P s}}{\tau_P s - 1} \tag{1}$$

where K_P, θ_P and τ_P are process gain, time delay, and time constant, respectively. The proposed feedback control structure is shown in Fig. 1 and the closed-loop response upon set-point change is

$$\left(\frac{C}{R}\right) = \frac{G_F G_C G_P}{1 + G_C G_P} \tag{2}$$

where G_F denotes the first-order set-point filter with the filtering time constant of τ_F. Upon substituting PI controller into the loop, Eq. (2) becomes

$$\left(\frac{C}{R}\right) = \frac{K_C\left(\frac{1}{\tau_F s+1}\right)\left(\frac{\tau_I s+1}{\tau_I s}\right)\left(\frac{K_P e^{-\theta_P s}}{\tau_P s-1}\right)}{1 + K_C\left(\frac{\tau_I s+1}{\tau_I s}\right)\left(\frac{K_P e^{-\theta_P s}}{\tau_P s-1}\right)} \tag{3}$$

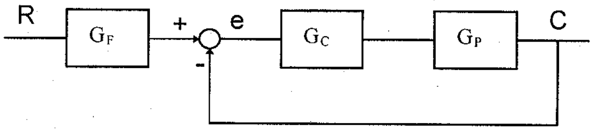

Fig. 1. Feedback control structure of the proposed method.

where K_C and τ_I are controller gain and integral time, respectively. Now we set

$$\tau_F = \tau_1 \tag{4}$$

which is always possible since both tuning parameters are at our disposal. The adoption of Eq. (4) is significant because it helps cancel the set-point filter term out of Eq. (3).

Then Eq. (3) becomes

$$\left(\frac{C}{R}\right) = \frac{K_C K_P e^{-\theta_P s}}{\tau_P \tau_I s^2 - \tau_I s + K_C K_P(\tau_I s + 1)e^{-\theta_P s}} \tag{5}$$

Next, the first-order Taylor expansion is employed to approximate the time delay term in the denominator to yield the following simple second-order closed-loop response.

$$\left(\frac{C}{R}\right) = \frac{e^{-\theta_P s}}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{6}$$

where

$$\tau = \sqrt{\tau_I \beta},\ \zeta = \frac{\tau_I - \theta_P - \tau_I/K_C K_P}{2\sqrt{\tau_I \beta}},\ \text{and}\ \beta = \frac{\tau_P}{K_C K_P} - \theta_P \tag{7}$$

Eqs. (6) and (7) show that we only need to determine the PI parameters, K_C and τ_I, provided that the process characteristics is known.

The Routh stability conditions about Eq. (6) yield

$$\beta > 0, \quad \text{and} \quad \tau_I - \theta_P - \tau_I/K_C K_P > 0,$$
$$\text{or} \quad \tau_I/K_P(\tau_I - \theta_P) < K_C < \tau_P/K_P\theta_P, \quad \text{and} \quad \tau_I > \theta_P \tag{8}$$

The above conditions can also be used to calculate the stability margins of the process once the values of the PI parameters are chosen.

3. Direct synthesis tuning method

Now the closed-loop response of Eq. (7) can be made the way we want by choosing properly the value of the damping ratio ζ. This is always possible with the suitable combination of K_C and τ_I of the PI controller as long as the unstable process is approximately modeled by a first-order-plus-time-delay system.

In our method we choose the value of unity for ζ in order to accomplish insuring overdamped nature of the closed-loop response while maintaining the maximum responsiveness of the controller at the same time. $\zeta < 1$ generates oscillation in response, and the critically damped response of $\zeta = 1$ represents the marginal case between overdamped and underdamped responses. Larger values than unity for ζ produce more sluggish responses than necessary. As for K_C, we choose the middle value from the allowed range given by the inequality of Eq. (8). The reason for this choice of K_C is that we want to keep maximum robustness margins for the both sides of the process parameters when the parameters get either larger or smaller than their estimated values.

The procedure to obtain the values of K_C and τ_I is as follows. First we assume a large value for τ_I, say, $100\tau_P$. Next we obtain the range of K_C as given by Eq. (8) and choose the middle point of this range as the value for K_C. Then using this value for K_C we solve the middle equation of Eq. (7) for the new τ_I with the unity value for ζ. We repeat this process to a convergence. [The golden section search method [8] was used for the solution of Eq. (7) after the equation being converted to have absolute values for the solutions.]

4. Simulation results

We have chosen as our example the unstable first-order-plus-time-delay system studied by Kavdia and Chidambaram [2] and have compared the performance of our proposed direct synthesis method with those of the other two methods, i.e. the conventional PI and the two-stage IMC.

$$G_P = \frac{e^{-0.5s}}{s-1} \tag{9}$$

The results of the parameter values obtained using the three methods are given in Table 1 and the corresponding process responses are shown in Fig. 2. We have used the same values of Kavdia and Chidambaram [2] for the conventional PI controller method and those of Jacob and Chidambaram [1] for the two-stage IMC method. It turns out that the performance of the conventional PI method is the worst while the performance of our direct synthesis method is as good as that of the best-tuned two-stage IMC method, i.e. the tuning parameter λ taking on the values of 2.0 and 2.5. Fig. 3 shows the comparison of the results when K_C is not taken from the middle point of the range as recommended by our proposed method, but rather 40, 60 and 70% of the range, which clearly demonstrates the middle point value produces the best. Fig. 4 shows the regulatory control results for a unit-step load change by our proposed DS method along with those by the IMC. The proposed method exhibits a fast regulatory control just as the two-stage IMC method does.

Next, since it has been known that when the process time delay, θ_P, is large, the first-order Taylor expansion of the time delay term may cause inaccurate computation for the tuning parameters of the system, we have checked this effect by examining the control performance of the system having different values of the process time delay. The results are shown in Table 2 and Fig. 5. (Here the ratio of θ_P/τ_P is kept at 0.5 in order to compare the results based on the same open-loop stability margin. Detailed explanation for the cases of different values of this ratio is given in the next section below.)

As the value of the time delay was increased up to 10 times that of the same process of Eq. (9), all responses in Fig. 5 exhibited similar smooth control performance. Of course as the time delay gets large, the response becomes sluggish because of the time constant which is also large due to the fixed ratio of θ_P/τ_P being 0.5. The result that the first-order Taylor expansion has no significant effect on the proposed tuning method can be explained by noting that the tuning parameters in

Table 1
Values of the tuning parameters by the different methods

	K_C	τ_I	$K_{C,i}$	τ_F	Tuning guideline
Conventional PI	1.4528	10.94			
Two-stage IMC	0.1402	0.9571	1.4142		$\lambda=2.0$
Two-stage IMC	0.1121	0.9571	1.4142		$\lambda=2.5$
Direct synthesis	1.5353	7.5753		7.5753	

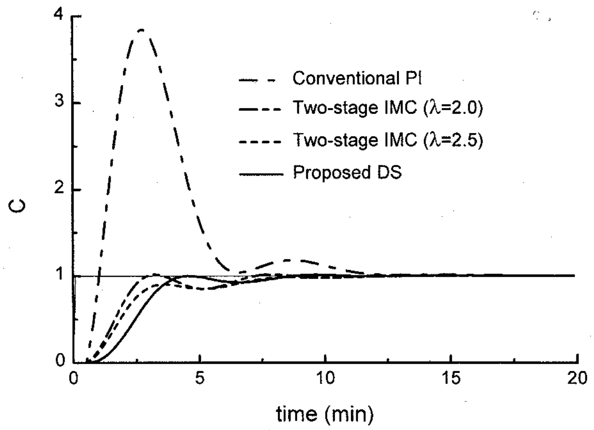

Fig. 2. Comparison of the step-change servo control responses.

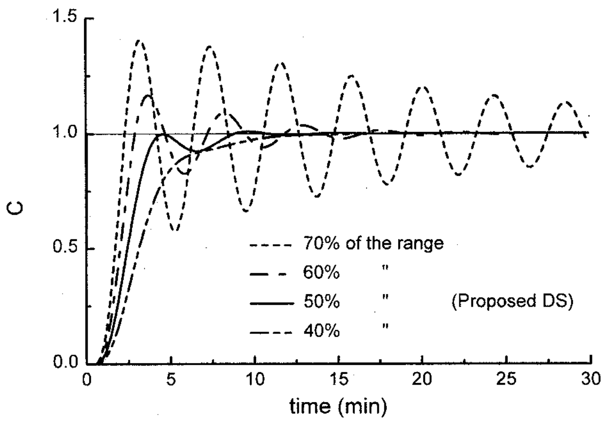

Fig. 3. Examples of the different K_C settings.

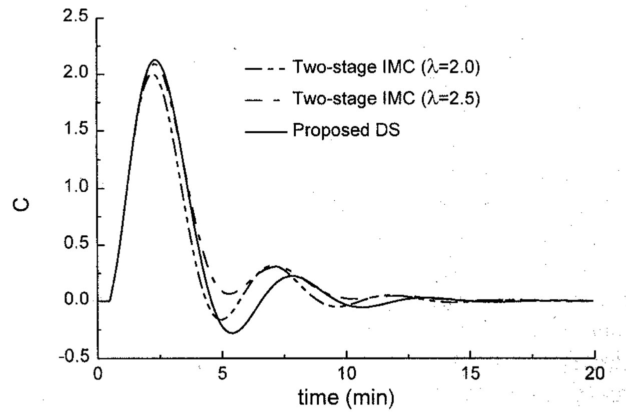

Fig. 4. Comparison of the unit-step load change regulatory control responses.

Table 2
Values of the tuning parameters for different values of time delay (θ_P/τ_P is kept at 0.5)

Process	K_C	τ_I	τ_F
$K_P=1.0$, $\tau_P=1.0$, $\theta_P=0.5$	1.5353	7.5753	7.5753
$K_P=1.0$, $\tau_P=4.0$, $\theta_P=2.0$	1.5353	30.3013	30.3013
$K_P=1.0$, $\tau_P=10.0$, $\theta_P=5.0$	1.5353	75.7533	75.7533

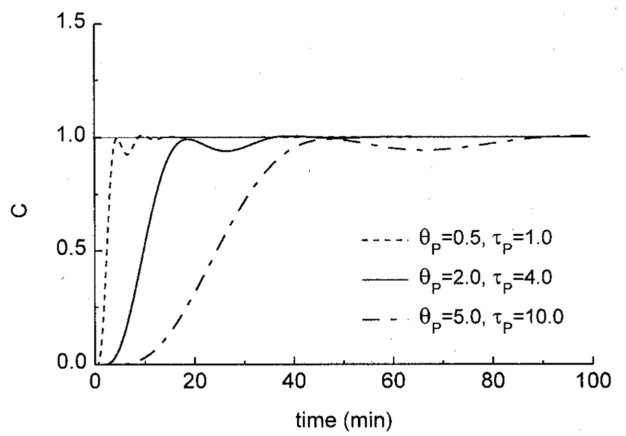

Fig. 5. Responses of the different time delay processes.

this method are chosen to maximize the stability margin for the critically damped response.

5. Stability Analysis

We now examine the effect of the ratio of θ_P/τ_P which was set at 0.5 in the above. The upper limit of this ratio in our system is 1.0 as seen from Eq. (8) which was based on the Routh stability criterion, because K_C is always greater than 1. The stability conditions corresponding to different values of the ratio have also been studied by other researchers before: De Paor and O'Malley [4] reported an optimal gain margin design method to find that the ratio should be less than 1 for the stable proportional control. Venkatashankar and Chidambaram [3] extended the stability condition to the proportional-integral control to obtain the closed-loop response is stable for θ_P/τ_P less than 0.775. [Their results were, though, presented for the ratio being 0.5 as in the example of Eq. (9).]

The results with different values of the ratio are shown in Table 3 and Fig. 6. As the value of the ratio θ_P/τ_P is increased, the controller gain, K_C, gets closer to 1 and the value of the integral time, τ_I, becomes extremely large. Also the range of K_C is narrowed down. All this is due to the fact that the system gets close to the marginal stability as the ratio of θ_P/τ_P approaches to 1, which then produces progressively more oscillatory response as shown in Fig. 6. The response of $\theta_P/\tau_P = 0.8$ then seems close to the marginal stability point under the proposed method, but it is still too oscillatory to apply in real processes, and so we here recommend our method to be applied to the processes of $\theta_P/\tau_P < 0.7$. The reason why this ratio is not the unity as given by the Routh stability condition of Eq. (8) is obviously that we have used an approximation of Taylor expansion in the derivation of the tuning method.

Table 3
Values of the tuning parameters for different values of θ_P/τ_P ratio

θ_P/τ_P	K_C	τ_I	τ_F	$K_{C,MIN}$	$K_{C,MAX}$
0.5	1.5353	7.5753	7.5753	1.0707	1.9999
0.6	1.3601	11.8164	11.8164	1.0535	1.6666
0.7	1.2333	19.0635	19.0635	1.0382	1.4285
0.8	1.1371	33.8155	33.8155	1.0243	1.2499
0.9	1.0613	78.5724	78.5724	1.0116	1.1111
0.99	1.0064	363.8178	363.8178	1.0028	1.0101

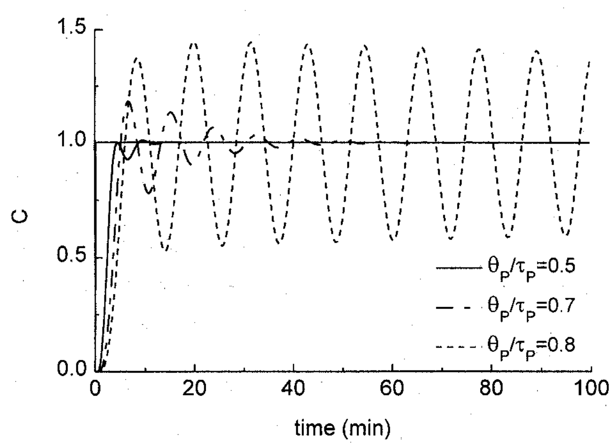

Fig. 6. Responses of the different θ_P/τ_P.

6. Robustness Test

The study of the controller tuning methods illustrated above cannot be complete without a robustness test: the performance of the controlled system should be checked when the values of the process parameters deviate from the estimated nominal values. We have investigated the effects of the uncertainties of the time delay, the time constant and the process gain, respectively. The results show satisfactory performance for all these three cases. Fig. 7 shows the case of the 20% increase of the time delay while Fig. 8 the case of the 20% increase of process gain. (The case of the 20% decrease of time constant

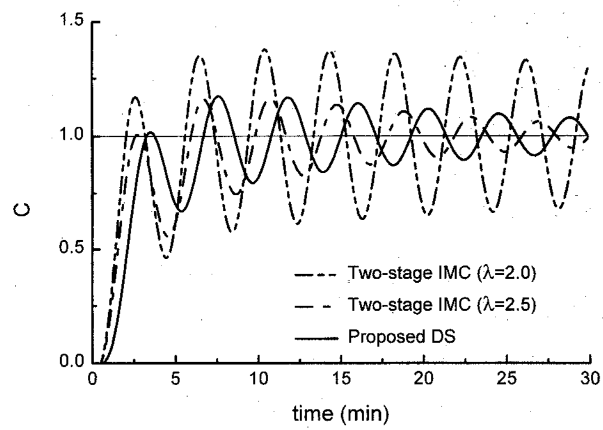

Fig. 7. Robustness test with the time delay increased by 20%.

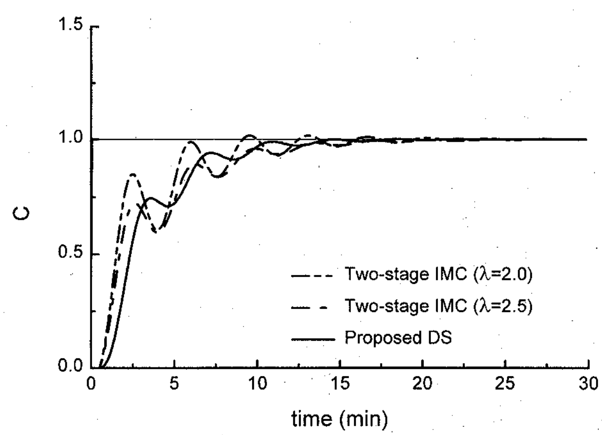

Fig. 8. Robustness test with the process gain increased by 20%.

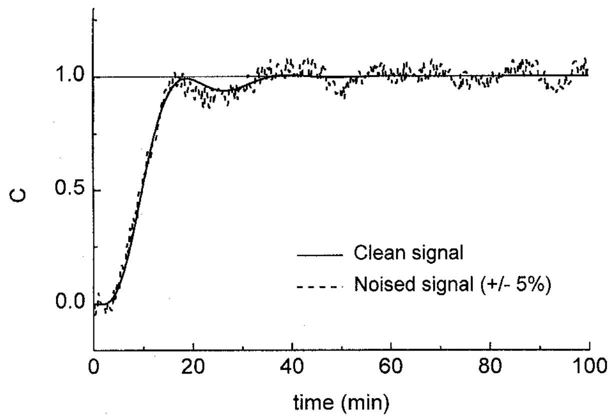

Fig. 9. Response with the random noise signal (± 5%).

shows a similar pattern as that of Fig. 8, since decreasing time constant and increasing time delay both make the system more unstable.) In all the cases, the direct synthesis method produces as good a performance as the best of the two-stage IMC with λ being equal to 2.5. (The case of λ equal to 2.5 performs better than λ of 2.0 in these robustness tests whereas the latter does better than the former in the step change response as shown in Fig. 2).

We also examined the robustness of the proposed tuning method against the signal noise. High frequency random noise signal was inserted into the process with ± 5% noise band compared to the set-point step change. The robust control performance can be obtained against the signal noise as shown in Fig. 9, due to the fact that there is no derivative control function in this proposed method.

7. Conclusions

For tuning the conventional PI controller, a direct synthesis method has been proposed using a first-order set-point filter whose time constant is set equal to that of the PI controller. The concept also incorporates both critical damping response and maximum robustness for the process parameters. The results prove that this easy-to-implement method produces as good performance as the best-tuned two-stage IMC method would have, with good robustness against the uncertainties of the process parameters as well as against the signal noise. Furthermore, while the selection of a proper value for λ in the IMC methods may not be easy without simulation study (due to its upper and lower limits), our proposed direct synthesis method doesn't need any tuning guidelines and contrasting with a cascade control structure adopted by the two-stage IMC method, it keeps the single feedback control structure resulting in smooth over-damped set-point tracking and good regulatory control for the first-order-plus-time-delay system. We believe that this simple feedback control structure also would help the plant operating personnel do their fine-tuning job more easily especially in the case of inaccurate process models.

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