Korea-Australia Rheology Journal
Vol. 11, No. 4, December 1999 pp.279-285

Draw resonance in polymer processing: a short chronology and a new approach

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Abstract

Draw resonance is both an important and interesting instability encountered in various extensional-deformation-dominated polymer processing operations. It is important because of its paramount relevance to the productivity and quality issue in the related industry, and it is interesting because of as yet unanswered questions as to what its cause and origin are in terms of physics involved. Specifically, a short chronological account of the draw resonance research is presented in this paper bringing several previous results together and focusing on the derivation of a new criterion for draw resonance based on the interaction of the traveling times of some kinematic waves propagating along the spinline from the die exit to the take-up position. The new explanation of draw resonance put forward here based on the physics of the system is seen to have wide implications on both theoretical and practical aspects of draw resonance instability. The importance of the role played by spinline tension in determining draw resonance is an example of the former whereas interpretation of the mechanism of the draw resonance eliminator is an example of the latter. Finally, an approximate yet a very fast and convenient method for determining draw resonance is also derived based on the above findings and found to agree well with the exact stability results.

1. Introduction

Draw resonance is a uniquely interesting phenomenon in many respects. First, it is easily observable in many polymer processing operations where extensional flow and deformation occur between the die exit and the take-up with the imposed boundary conditions of fixed take-up velocity. Second, it is an instability easily determinable by traditional linear stability analysis tools. Third, it is, however, not yet amenable to any easy interpretation based on the physics involved as to the fundamental nature of its origin and cause.

As regards the first two aspects above, there are plenty of reports on the research efforts rendered toward elucidating this phenomenon during the past four decades. Ever since the draw resonance was first observed and aptly named as such by Christensen (1962) and Miller (1963), there have been many experimental observations and theoretical attempts to explain this seemingly simple phenomenon (Pearson and Matovich, 1969; Gelder, 1971; Donnelly and Weinberger, 1975; Fisher and Denn, 1976; Ishihara and Kase, 1976; White and Ide, 1978; Hyun 1978; Kase and Araki, 1982; Lucchesi *et al.*, 1985; Anturkar and Co, 1988, Cain and Denn, 1988; Liu and Beris, 1988; Kim *et al*, 1996a, 1996b; Jung and Hyun, 1999, 1999a, etc.). The reason for this apparent plethora is basically two-fold. On one side, the issue of productivity and quality which is always of paramount importance industrially is, in many continuous polymer processing operations like fiber spinning, film casting and film blowing, profoundly influenced by the draw resonance instability. On another side, the academic researchers' interests are usually centered on the subjects like stability, sensitivity and optimization of the process which draw resonance in polymer processing operations is all about.

While experimental results on draw resonance continuously had come out from the various laboratories (Donnelly and Weinberger, 1975; Ishihara and Kase, 1976; White and Ide, 1978), the theoretical attempts also kept pace starting with Japanese modeling efforts (Kase and Matsuo, 1965) and the first stability analysis by a British group (Pearson and Matovich, 1969) using simple Newtonian models. Employment of linear stability analysis after these two pioneering efforts came along beginning with Gelder's successful results (Gelder, 1971). Interests in draw resonance were pursued further along with modeling on extensional flows of both Newtonian and viscoelastic fluids. Of notable among many those research reports are Denn *et al.* (1975), and Bechtel *et al.* (1992) being the first and most recent successful modeling of viscoelastic fluids for spinning, respectively, and Fisher and Denn (1976) reporting on the stability results using the said models. Petrie and Denn (1976) and Larson (1992) are the two comprehensive reviews, one in 70s and the other in 90s, on the whole gamut of polymer processing instabilities includ-

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ing draw resonance. The book by Petrie (1979) on extensional flows was a timely, valuable contribution to the research of the whole field while Petrie (1988) elucidated many relevant, unresolved points involved in the stability of extensional flows including those in draw resonance.

Despite all these efforts and successful endeavors toward the research on draw resonance in terms of modeling and stability determination, there remained the fundamental questions still left, i.e., why the draw resonance phenomenon does occur and what the physics behind is. This is partly due to the fact that linear stability analysis only produces the critical conditions at the onset of instability without necessarily revealing the fundamental cause of instability itself, and thus during the course of computation of the eigenvalues of the stability matrix the physics of the system as related to stability is usually lost. In an effort to gain some understanding on the physics of the system, investigation was carried out into the dynamic behavior of propagating waves on the spinline which are the direct results of the hyperbolic nature of the governing equations of the spinning systems. Hyun (1978) elaborated the mechanism of how the disturbances on the spinline transmit and repeat themselves from the take-up to the spinneret via the spinline tension resulting in the perpetuation of draw resonance. Beris and Liu (1988) clearly demonstrated and explained the hyperbolic nature of the system for spinning and Kase and Araki (1982) showed the causality between disturbances and process in spinning using a linear transfer function approach. Kim *et al.* (1996a) derived a criterion for draw resonance based on traveling times of waves on the spinline. The ingenious device called draw resonance eliminator developed by Lucchesi *et al.* (1985) at Union Carbide also exploits the same kind of the idea, which represents a shining example how industrial R & D goes ahead of academic ones in putting basic, fundamental ideas to work for realizing enhanced process productivity.

In this paper, we make a short chronology on draw resonance research along the lines described above by bringing several previous results together to shed light further on what kind of physics is involved in this industrially and academically important phenomenon. By so doing we also intend to provide some useful perspectives on the practical issues of productivity and quality for the related industrial processes.

2. Linear stability analysis

As mentioned in the previous section, conventional linear stability analysis method was successfully applied by many researchers to spinning processes to produce the critical values of the drawdown ratio at the onset of draw resonance instability. By determining the sign and magnitude of the largest eigenvalues of the stability matrix which is derived linearizing governing equations and introducing transient perturbation to the dependent variables, we can always find the stability/instability regions in the chosen parameter space. In other words, as long as the model equations of the system are available, the stability of the system is thus readily determined. As an example, a simple case of stability analysis is presented in the following for the isothermal spinning of Maxwell fluids as reported by Jung and Hyun (1999).

Continuity equation:

$$\frac{\partial A}{\partial t}+\frac{\partial (AV)}{\partial x}=0 \qquad (1)$$

Equation of motion:

$$\frac{\partial}{\partial x}[A(\tau_{xx}-\tau_{rr})]=0 \qquad (2)$$

Constitutive equation:

$$\tau_{xx}\left(1+\bar{a}\sqrt{3}De\frac{\partial V}{\partial x}\right)+De\left(\frac{\partial \tau_{xx}}{\partial t}+V\frac{\partial \tau_{xx}}{\partial x}-2\tau_{xx}\frac{\partial V}{\partial x}\right)=2g\frac{\partial V}{\partial x}$$

$$\tau_{rr}\left(1+\bar{a}\sqrt{3}De\frac{\partial V}{\partial x}\right)+De\left(\frac{\partial \tau_{rr}}{\partial t}+V\frac{\partial \tau_{rr}}{\partial x}-\tau_{rr}\frac{\partial V}{\partial x}\right)=-g\frac{\partial V}{\partial x} \qquad (3)$$

These equations are subject to the following boundary conditions.

$$A=A_0=1,\ V=V_0=1,\ \tau_{xx}=\tau_0 \text{ at } x=0 \text{ for all } t$$

$$V=V_L=r \qquad \text{at } x=1 \text{ for all } t \qquad (4)$$

where A = dimensionless spinline cross-sectional area, V = dimensionless spinline velocity, τ_{xx} = dimensionless spinline axial stress, τ_{xx} = dimensionless spinline radial stress, x = dimensionless distance coordinate from the spinneret, t = dimensionless time, g = dimensionless reciprocal axial force, De = Deborah number, $\bar{a}$ = parameter representing the strain-rate dependency of material relaxation times, r = drawdown ratio. Subscripts 0, L denote spinneret and take-up conditions, respectively.

At time $t=0^+$, disturbances are introduced to the system with all other conditions held the same.

In the above, several assumptions have been incorporated. First, the variations of variables across the spinline cross-section are neglected to result in an one-dimensional model for the system. Second, the origin of the spinning distance coordinate is chosen at the die (extrudate) swell position ignoring the pre-spinneret conditions on the spinline. Third, all the secondary forces on the spinline, i.e., gravity, air drag, surface tension and inertia are neglected.

Now transient perturbations are introduced to the dependent variables as follows.

$$A(t,x)=A_s(x)+\alpha(x)e^{\Omega t},\quad V(t,x)=V_s(x)+\beta(x)e^{\Omega t}$$

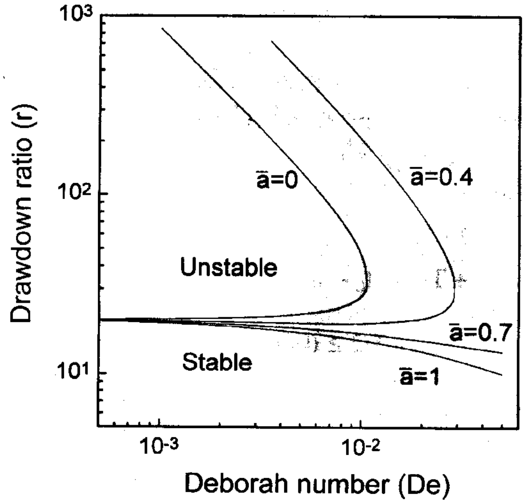

Fig. 1. Stability diagrams of various convected Maxwell fluids by linear stability method.

$$\tau_{xx}(t, x) = \tau_{xx,s}(x) + \gamma(x)e^{\Omega t}$$

$$\tau_{xx}(t, x) - \tau_{rr}(t, x) = (\tau_{xx,s}(x) - \tau_{rr,s}(x)) + \delta(x)e^{\Omega t} \qquad (5)$$

where, the subscripts S indicates steady state, and α, β, γ and δ are the perturbed quantities, and Ω is a complex eigenvalue that accounts for the growth rate of the perturbation.

The stability diagram in Fig. 1 has been recalculated from Jung and Hyun (1999) where the detailed derivation was explained for the linear stability analysis. The same analysis can of course be performed for other more complex spinning systems. For example, the cases of secondary forces included, nonisothermal spinning and other constitutive equations could be similarly dealt with to yield similar stability results. It can be reiterated here that the linear stability analysis readily produces the stability results as long as the proper governing equations of the system are available for linearization and perturbations. The remaining questions are, however, why this draw resonance instability does occur and what physics is involved to cause it.

3. Physics behind the draw resonance phenomenon

In an effort to shed some light on the physics involved in causing draw resonance in fixed take-up velocity spinning, transient simulation was performed on the system to produce pictures showing transient behavior of the spinline variables as depicted in Fig. 2. The three dimensional transient pictures of the spinline cross-sectional area (A) wave and the throughput (AV) wave of the case of Fig. 2 are shown in Figs. 3 and 4, respectively. Due to the hyperbolic nature of the governing equations of the system described by Eqs. (1)~(4), any disturbances tend to propagate in the form of waves along the spinline as the fluid elements travel from

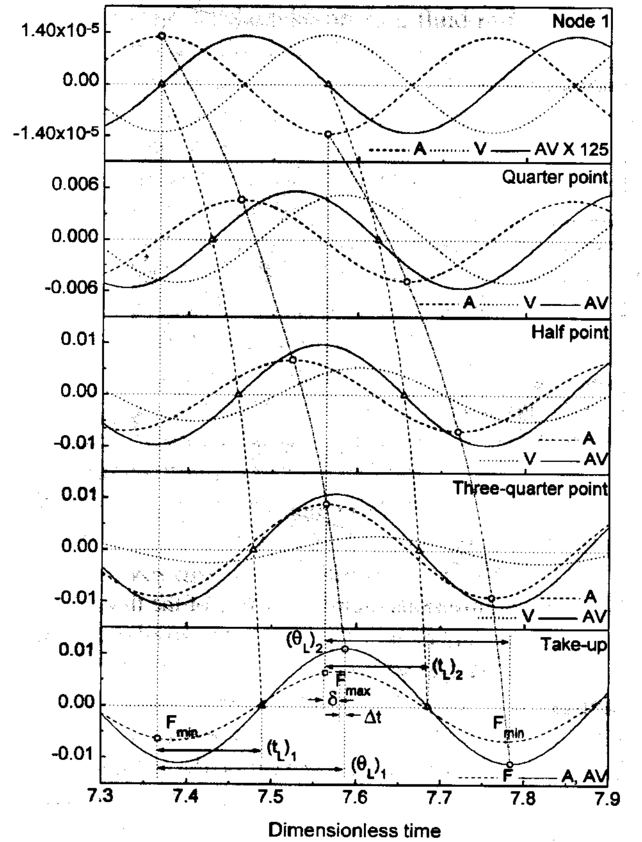

Fig. 2. Transient behavior of spinline variables of Maxwell fluid at five different spatial positions of the spinline when $\bar{a}$ = 0.4, De = 0.019, r = r_C = 27.97.

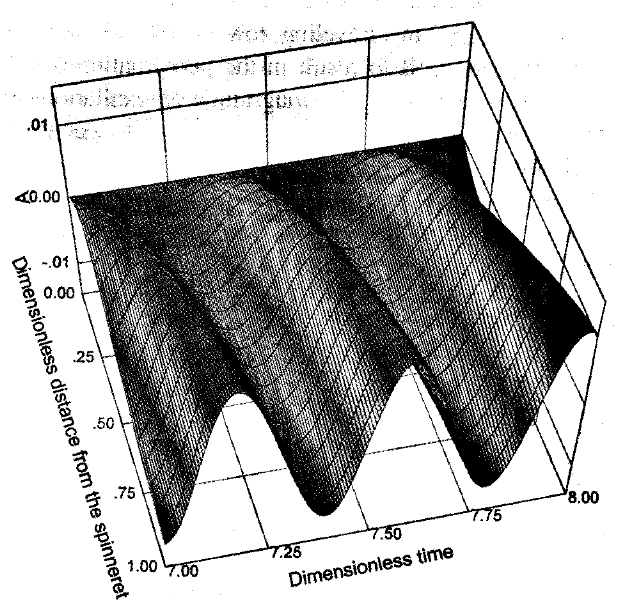

Fig. 3. Three-dimensional transient picture of the spinline cross-sectional area (A) wave.

the spinneret to the take-up. Also since the take-up velocity is maintained at a fixed value, any disturbance waves continuously generate another waves at the spinneret as

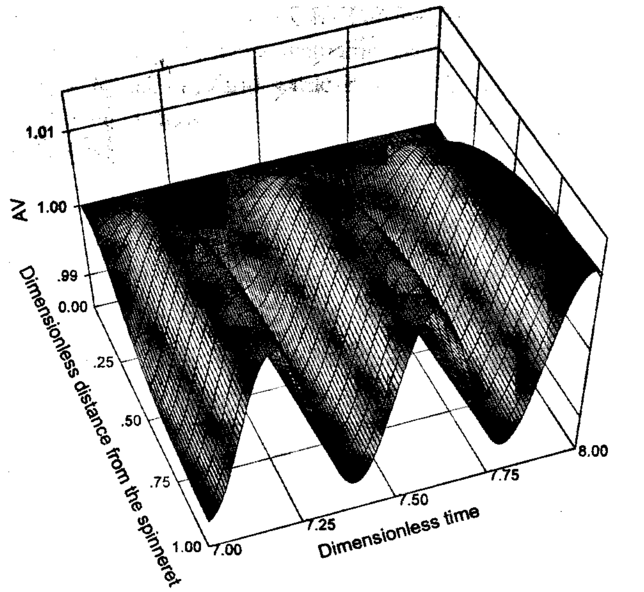

Fig. 4. Three-dimensional transient picture of the throughput (AV) wave.

soon as they exit the system through the take-up position. Here the important linkage which makes possible continuous generation of the waves is the spinline tension which transmits the conditions at the take-up to the spinneret in an instantaneous fashion for Newtonian fluids spinning and with a small delay for viscoelastic fluids spinning. In other words, as one wave goes out of the system through the take-up, another similar wave (of different sign of magnitude) immediately appears at the spinneret and begins traveling toward the take-up. This process repeats itself to result in the perpetuation of draw resonance phenomenon. The magnitude of oscillations dies out with time in stable situations whereas it reaches the steady levels of limit cycles in unstable situations. The explanations along this line was also presented by Kim *et al.* (1996a), Jung and Hyun (1999), Jung *et al.* (1999a), and Jung *et al.* (1999b).

This is the physical picture of draw resonance in which nonlinear oscillation of spinline variables such as cross-sectional area, tension and throughput evolves with time. It was found by Hyun (1978) that the hyperbolic nature of the continuity equation of spinning process causes throughput waves to travel the spinline. Particularly, among them the unity throughput waves travel the entire spinning distance from the spinneret to the take-up and play a key role in deciding the onset conditions of draw resonance as shown by Kim *et al.* (1996a) where other waves were also found to travel on the spinline with their uniquely different traveling velocities.

4. Criterion for draw resonance

As explained above, there are many different traveling times corresponding to different waves in the system. Now we seek to find some relationships among these traveling times in connection with the onset of draw resonance. It was found by Kim *et al.* (1996a) that the following important relationship exists among these different traveling times and the sign of the inequality changes according as the stability of the system changes.

$$(t_L - \Delta t)_1 + T/4 + (t_L - \Delta t)_2 + T/4 \geq [\theta_L - \Delta t]_1 + [\theta_L - \Delta t]_2 = T,$$

$$\text{or } (t_L)_1 + (t_L)_2 + T/2 \geq (\theta_L)_1 + (\theta_L)_2, \quad \text{for } r \leq r_C \qquad (6)$$

where, t_L = dimensionless traveling time of unity-throughput waves from the spinneret to the take-up, Δt = dimensionless time (phase) differences between the spinline tension curve and spinline area curve at the take-up, T = dimensionless period of oscillation, θ_L = dimensionless traveling time of maximum area or minimum area waves from node 1 to the take-up, and r_C = critical drawdown ratio at the onset of draw resonance.

The onset case of draw resonance of Fig. 2 is redrawn in Fig. 5 where the relation of Eq. (6) is readily seen.

This relation is thus the criterion for draw resonance where the equality represents the case of the onset of draw resonance. Fig. 6 clearly shows this criterion for draw resonance for the spinning case of Eqs. (1)-(4) which is in exact agreement with the values obtained by the linear stability analysis method.

Interpretation of this criterion of draw resonance is as follows. The left hand side of Eq. (6) is the required time for two successive unity throughput waves (with a pause time of T/4 in between) to be able to travel from the spinneret to the take-up whereas the right hand side is the time for two successive peak and trough cross-sectional area waves travel the same spinning distance. This right hand side is also equal to the period of oscillation. What

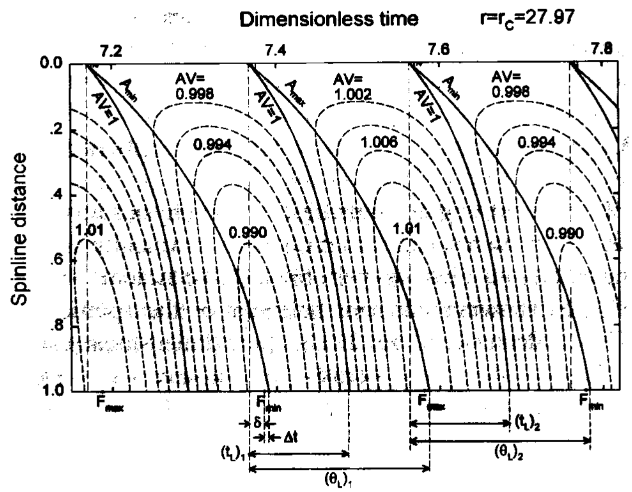

Fig. 5. Contours of constant-AV curves when $\bar{a}$ = 0.4, De = 0.019, r = r_C = 27.97.

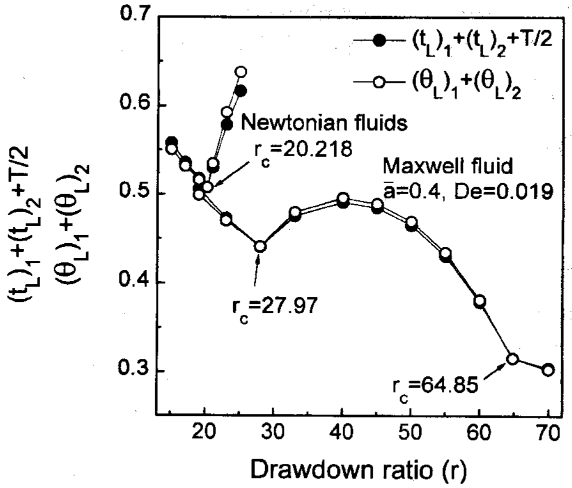

Fig. 6. Comparison of traveling times of throughput waves and cross-sectional area waves as plotted against the drawdown ratio.

Eq. (6) says is that if the drawdown ratio is smaller than its critical value, i.e., stable situations, the required time for waves on the left side becomes greater than the allowed time by the two peak area waves on the right side, which makes sustained oscillation impossible. When the drawdown ratio is at the critical value, these two times of the left and right sides become equal to each other just enabling to sustain steady oscillation. If the drawdown ratio becomes larger than its critical value, the required time is always smaller than the allowed time so that steady sustained oscillation of draw resonance is possible.

The criterion for draw resonance represented by Eq. (6) thus clearly determines when the onset of draw resonance actually occurs. This is a hydrodynamic condition for the spinning system and thus applies to any process conditions involving various constitutive equations and spinline cooling conditions. In other words, graphs similar to Fig. 6 are always possible for any spinning systems to determine the occurrence of draw resonance as the drawdown ratio is increased from the stable to unstable ranges.

5. Approximate method for stability determination

There is another utility for the draw resonance criterion of Eq. (6). As Jung *et al.* (1999a) explained, Eq. (6) can be approximated using the fluid element traveling time, i.e., fluid residence time on the spinline and an approximate throughput traveling times. Putting together these two approximate times for Eq. (6), Jung *et al.* (1999a) came up with an approximate criterion for draw resonance as shown in Eq. (7).

$$2\left(\frac{\ln r}{r-1}\right)\approx\tau_L \qquad (7)$$

where, τ_L = dimensionless traveling time of fluid elements from the spinneret to the take-up, i.e., fluid residence time on the spinline.

The utility of this approximate criterion for determining the onset of draw resonance is rather great because using this criterion, we don't need to solve the transient differential equations of the system, but rather use steady state solution only which is easily obtained. As for the question of how accurate this approximation is, Jung *et al.* (1999a) and Kim *et al.* (1996b) showed that the agreement is qualitatively good for film casting, film blowing, and fiber spinning, respectively. Figs. 7, 8 and 9 show the examples of these fiber spinning, film casting and film blowing cases, respectively, where the results by this approximate method are compared with the exact results recalculated by linear stability analysis method of Jung and Hyun (1999), Anturkar and Co (1988), and Cain and Denn (1988), respectively.

6. Perspectives and Conclusions

So far we have presented a new criterion for draw resonance based on the traveling times of spinline waves contributing to a better understanding of the phenomenon in terms of its cause and mechanism. The results can be applied to any extensional deformation processes such as fiber spinning, film casting and film blowing to improve their industrially important process productivity and product quality. The subjects like stability, sensitivity and optimization can also be tackled for these processes using the same concept. For example, the effects of various process conditions including fluid properties on draw resonance can be analyzed for the benefits of process productivity improvement. Specifically, the effects of process conditions which have not been considered in this paper such as spinline secondary forces, spinline cooling, and fluid viscoelasticity, etc. on the dynamics of the process

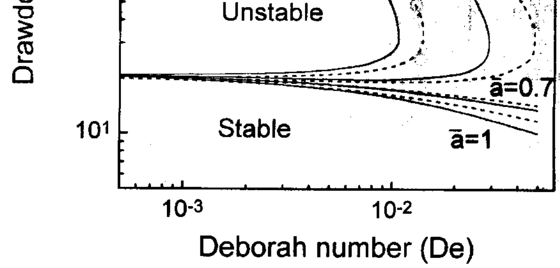

Fig. 7. Comparison of stability results between the exact and approximate methods for fiber spinnning process.

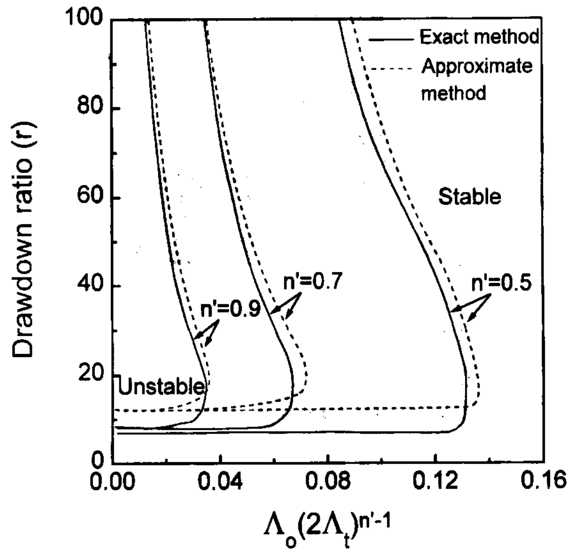

Fig. 8. Comparison of stability results between the exact and approximate methods for film casting process.

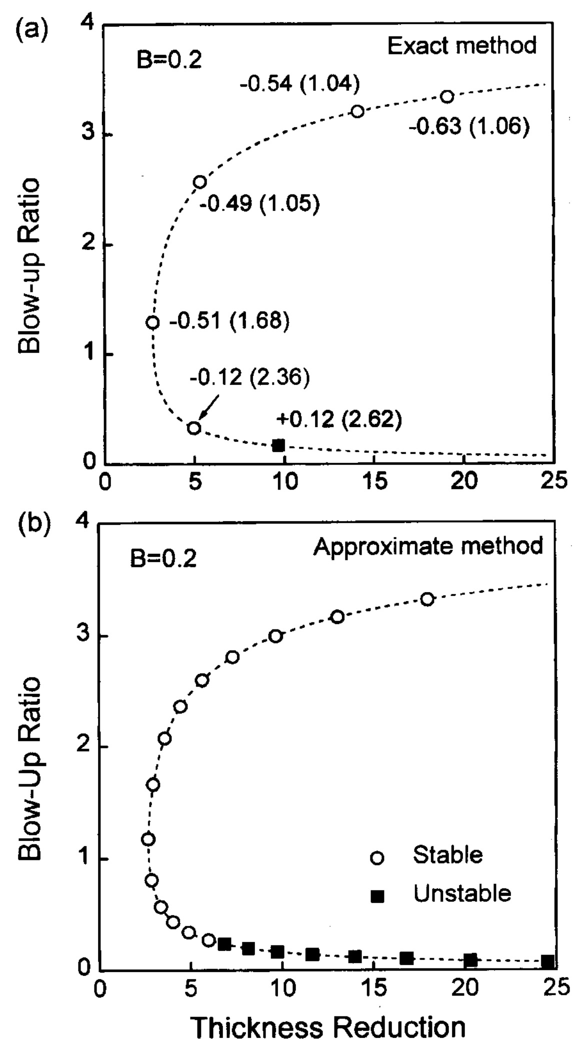

Fig. 9. Comparison of stability results between (a) the exact and (b) approximate methods for tubular film blowing process.

including draw resonance can be easily studied, e.g., Jung *et al.* (1999c), following the procedure described here. By doing so, we can delve into many important subjects of the process dynamics surrounding draw resonance as expounded by Petrie (1988).

Acknowledgments

Financial assistance by the Korean Science and Engineering Foundation (KOSEF) under the Grant 1999-1-308-001-3 to this study is greatly appreciated.

References

Anturkar, N.R. and A. Co, 1988, "Draw resonance in film casting of viscoelastic fluids: A linear stability analysis," *J. Non-Newt. Fluid Mech.* **28**, 287.

Bechtel, S.E., J.Z. Cao, and M.G. Forest, 1992, "Practical application of a higher order perturbation theory for slender viscoelastic jets and fibers," *J. Non-Newt. Fluid Mech.* **41**, 201.

Beris, A.N. and B. Liu, 1988, "Time-dependent fiber spinning equations. 1. Analysis of the mathematical behavior," *J. Non-Newt. Fluid Mech.* **26**, 341.

Cain, J.J. and M.M. Denn, 1988, "Multiplicities and instabilities in film blowing," *Polym. Eng. Sci.* **28**, 1527.

Christensen, R.E., 1962, "Extrusion coating of polypropylene," *S. P. E. J.* **18**, 751.

Denn, M.M., C.J.S. Petrie, and P. Avenas, 1975, "Mechanics of steady spinning of a viscoelastic liquid," *AIChE J.* **21**, 791.

Donnelly, G.J. and C.B. Weinberger, 1975, "Stability of isothermal fiber spinning of a Newtonian fluid," *Ind. Eng. Chem. Fundam.* **14**, 334.

Fisher, R.J. and M.M. Denn, 1976, "A theory of isothermal melt spinning and draw resonance," *AIChE J.* **22**, 236.

Gelder, D., 1971, "The stability of fiber drawing processes," *Ind. Eng. Chem. Fundam.* **10**, 534.

Hyun, J.C., 1978, "Theory of draw resonance: I. Newtonian fluids," *AIChE J.* **24**, 418; also, "Part II. Power-law and Maxwell fluids," **24**, 423.

Ishihara, H. and S. Kase, 1976, "Studies on melt spinning. VI. Simulation of draw resonance using Newtonian and power law viscosities," *J. Appl. Polym. Sci.* **20**, 169.

Jung, H.W. and J.C. Hyun, 1999, "Stability of isothermal spinning of viscoelastic fluids," *Korean J. Chem. Eng.* **16**, 325.

Jung, H.W., S.M. Choi, and J.C. Hyun, 1999a, "An approximate method for determining the stability of film casting process," *AIChE J.* **45**, 1157.

Jung, H.W., H.-S. Song, and J.C. Hyun, 1999b, "Draw resonance and kinematic waves in the isothermal melt spinning of viscoelastic fluids," *AIChE J. being reviewed.*

Jung, H.W., H.-S. Song, and J.C. Hyun, 1999c, "Analysis of the stabilizing effect of spinline cooling in melt spinning," *J. Non-Newt. Fluid Mech.* **87**, 165.

Kase, S. and T. Matsuo, 1965, "Studies on melt spinning. I. Fundamental equations on the dynamics of melt spinning," *J. Polym. Sci., Part A.* **3**, 2541.

Kase, S. and M. Araki, 1982, "Studies on melt spinning. VIII. Transfer function approach," *J. Appl. Polym. Sci.* **27**, 4439.

Kim, B.M., J.C. Hyun, J.S. Oh, and S.J. Lee, 1996a, "Kinematic waves in the isothermal melt spinning of Newtonian fluids," *AIChE J.* **42**, 3164.

Kim, B.M., S.M. Choi, H.W. Jung, and J.C. Hyun, 1996b, "An approximate method for determining the stability of film casting, film blowing, and fiber spinning," Australia-Korea Workshop on Polymer Melt and Polymer Solution Rheology, Univ. of Melbourne, Parkville, Victoria, Australia, 103.

Larson, R.G., 1992, "Instabilities in viscoelastic flows," *Rheol. Acta* **31**, 213.

Liu, B. and A.N. Beris, 1988, "Time-dependent fiber spinning equations. 2. Analysis of the stability of numerical approximations," *J. Non-Newt. Fluid Mech.* **26**, 363.

Lucchesi, P.J., E.H. Roberts, and S.J. Kurtz, 1985, "Reducing draw resonance in LLDPE film resins," *Plast. Eng. May* 87.

Miller, J.C., 1963, "Swelling behavior in extrusion," *S. P. E. Trans.* **3**, 134.

Pearson, J.R.A. and M.A. Matovich, 1969, "Spinning a molten threadline: Stability," *Ind. Eng. Chem. Fundam.* **8**, 605.

Petrie, C.J.S. and M.M. Denn, 1976, "Instabilities in polymer processing," *AIChE J.* **22**, 209.

Petrie, C.J.S., 1979, Elongational Flows, Pitman Publishing, London.

Petrie, C.J.S., 1988, "Some remarks on the stability of extensional flows," *Progress and Trends in Rheology II*, 9.

White, J.L. and Y. Ide, 1978, "Instabilities and failure in elongational flow and melt spinning of fibers," *J. Appl. Polym. Sci.* **22**, 3057.