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Simplified modeling of slide-fed curtain coating flow

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Abstract

Simplified model of slide-fed curtain coating flow has been developed and tested in this study. It rests on the sheet profile equations for curtain thickness in curtain flow and its trajectory derived by the integral momentum balance approach of Higgins and Scriven (1979) and Kistler (1983). It also draws on the film profile equation of film thickness variation in flow down a slide. The equations have been solved in finite difference approximation by Newton iteration with continuation. The results show that how inertia (Reynolds number), surface tension (capillary number), inclination angle of the slide, and air pressure difference across the curtain affect sheet trajectory and thickness profile. It has been revealed that approximate models can be useful to easily analyze coating flow dynamics without complex computations, giving qualitative agreement with full theory and with experiment.

Keywords **:** curtain coating, slide flow, curtain flow, sheet profile equation, curtain trajectory, integral momentum balance, simplified modeling

1. Introduction

Curtain coating, in its precision mode, is a premetered coating process that has been used to manufacture single-layer and, most notably, multilayer coatings and patch coatings by falling freely on substrates or webs moving at relatively high speeds (e.g., faster than 5 m/s). The curtain can be delivered by a slot die, as in casting of polymeric sheet, or, if the number of layers exceeds two or three, by a slide die (Miyamoto and Katagiri, 1997).

In the slide-fed curtain coating that would be investigated in this study, the flow can be divided by the several sub-regions (Fig. 1) to systematically analyze the flow dynamics of this process; film flow region on the inclined slide, curtain forming region around the slide lip where the liquid changes its direction, curtain flow region beyond the lip where the falling liquid experiences uniaxial extensional deformation by gravity force, impingement region where falling liquid impacts the moving substrate, and take-away region where liquid attains fully developed plug flow with the substrate speed.

What is unique to this process, as mentioned above, is the unconstrained liquid sheet that falls freely one to many centimeters before impinging on the substrate being coated. In general, the sheet is subject to more instabilities (Lin, 1981), for example, the periodic oscillations of "draw resonance" in viscous curtains (Yeow, 1974), and is more susceptible to external disturbances, e.g., air pressure variations (Weinstein *et al*., 1997), than other coating operations. Therefore, computer-aided theoretical modeling is valuable in understanding, predicting, and controlling curtain behavior.

The excellent analysis of curtain coating process has

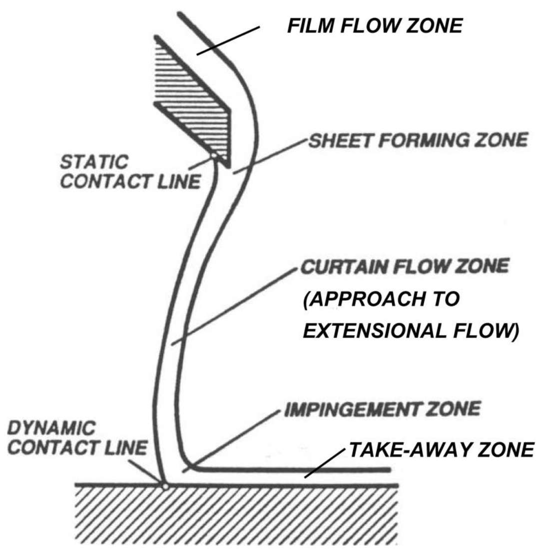

Fig. 1. Schematic of the slide-fed curtain coating flow.

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been pioneered by Kistler (1983) and followed by Ogawa and Scriven (1990) by means of the two-dimensional Navier-Stokes modeling for viscous free surface flow (Clarke, 1968). Also, by the same means Katagiri (1992) probed the instability of this process and Oki and Scriven (2002) recently explored the instability of multilayer curtain coating system. Such theoretical analysis is so challenging and time-consuming that a simpler approximate model would be valuable, once its range of validity were known by comparison with fuller theory and with experiment. Approximate models of this sort have proven useful in film casting (Yeow, 1974; Jung *et al.*, 1999; Lee *et al.*, 2003), slot coating (Higgins and Scriven, 1979; Cohen 1993), slide coating (Nagashima, 1993), roll coating (Coyle, 1984), and other coating flows.

To shed light on the curtain flow dynamics, Finnicum *et al*. (1983) theoretically and experimentally examined the two-dimensional inviscid planar curtain subjected to an applied pressure drop. Weinstein *et al*. (1997) also investigated two-dimensional liquid curtain flow from slot die assuming the potential flow. As such efforts to explicitly analyze the curtain flow, this study has been extended to approximately set up governing model for the curtain flow combined with the slide flow. It rests on one-dimensional sheet profile equations for curtain thickness and trajectory based on the integral momentum balance approach of Higgins and Scriven (1979) and Kistler (1983); it also draws on the one-dimensional film profile equation of film thickness variation in flow down a slide. Curtain-forming region between two regions was roughly approximated using simple matching conditions.

2. Simplified model of the slide flow

The full Navier-Stokes system shows the following dimensionless governing equations in slide flow region. The configuration of the slide flow is depicted in Fig. 2a.

Equation of continuity (EOC):

$$\underline{\nabla}\cdot\underline{U}=\underline{0} \tag{1}$$

where $\underline{U}\equiv\dfrac{\underline{U}^{*}}{(q/h_0)}$, $h_0\equiv\left(\dfrac{3\mu q}{\rho g\sin\theta}\right)^{1/3}$, $\underline{\nabla}\equiv h_0\underline{\nabla}^{*}$

Equation of motion (EOM):

$$Re\underline{U}\cdot\underline{\nabla}\underline{U}=\underline{\nabla}\cdot\underline{\underline{T}}+St\,\underline{f} \tag{2}$$

where $Re\equiv\dfrac{\rho q}{\mu}$, $\underline{\underline{T}}\equiv\dfrac{\underline{\underline{T}}^{*}}{(\mu q/h_0^2)}=-P\underline{\underline{I}}+(\underline{\nabla}\underline{U}+\underline{\nabla}\underline{U}^{T})$, $St\equiv\dfrac{\rho g h_0^3}{\mu q}$,

$\underline{f}=\sin\theta\underline{i}-\cos\theta\underline{j}$

Stress boundary condition at free surface:

$$\underline{n}\cdot\underline{\underline{T}}=-\underline{n}P_a+\frac{1}{Ca}\frac{d\underline{t}}{ds_f} \tag{3}$$

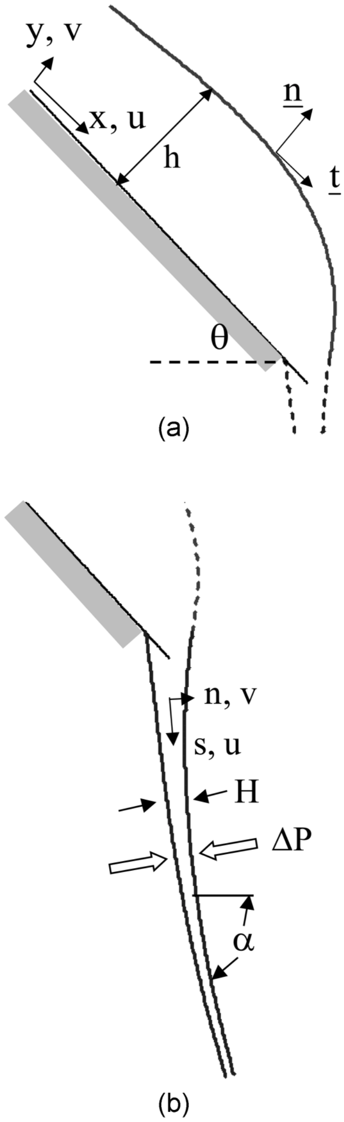


Fig. 2. Simplified flow geometries of (a) the slide flow and (b) the curtain flow.

where $\underline{n}\equiv\dfrac{-\partial h/\partial x\underline{i}+\underline{j}}{\sqrt{(\partial h/\partial x)^2+1}}$, $\underline{t}\equiv\dfrac{\underline{i}+(\partial h/\partial x)\underline{j}}{\sqrt{(\partial h/\partial x)^2+1}}$, and $Ca\equiv\dfrac{\mu q}{\sigma h_0}$

Kinematic boundary condition at free surface:

$$\underline{n}\cdot\underline{U}=0 \tag{4}$$

No-slip condition at solid surface: $\underline{U}=\underline{0}$ (5)

where $\underline{U}(=u\underline{i}+v\underline{j})$ is the velocity vector, q volumetric flow rate per unit film width, $\underline{\underline{T}}$ total stress tensor, $\underline{f}$ body force vector, P_a ambient pressure, θ slide inclination angle from the horizontal line, *Re* Reynolds number, *St* stokes number, *Ca* capillary number, h film thickness, h_0 fully developed film thickness flowing down the slide (characteristic length scale), ρ liquid density, μ liquid viscosity, s_f arc length, x and y dimensionless space coordinates parallel and perpendicular to the slide, $\underline{n}$ and $\underline{t}$ normal and tangential unit vector at the free surface, and $\underline{i}$ and $\underline{j}$ tangential and normal unit vector at the slide, respectively. Superscript * denotes the dimensional property.

The governing equations in the slide flow and also curtain flow can be simplified by means of lubrication approximation through integral momentum balance approach (Higgins and Scriven, 1979), provided that both flow regimes are nearly rectilinear. On the assumption that velocity component normal to the slide surface, v, is small and slowly changing, above governing equations are simplified in sequence as follows.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{(EOC)} \tag{1a}$$

$$Re\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\frac{\partial^2 u}{\partial y^2}+St\sin\theta \quad \text{(x-comp. EOM)} \tag{2a}$$

$$0=-\frac{\partial P}{\partial y}-St\cos\theta \quad \text{(y-comp. EOM)} \tag{2b}$$

$$P+\frac{\kappa}{Ca}=0 \quad \text{at } y=h \text{ (normal stress BC)} \tag{3a}$$

where $\kappa \equiv \underline{n}\cdot\frac{d\underline{t}}{ds_f}=\frac{(d^2h)/dx^2}{(1+(dh/dx)^2)^{3/2}}$ (= curvature)

$$\frac{\partial u}{\partial y}=0 \quad \text{at } y=h \text{ (tangential stress BC)} \tag{3b}$$

$$v=\frac{\partial h}{\partial x}u \quad \text{at } y=h \text{ (kinematic BC)} \tag{4a}$$

$$u=0,\ v=0 \quad \text{at } y=0 \text{ (no-slip BC)} \tag{5a}$$

In this case, ambient pressure, P_a, is negligible compared to the liquid pressure. Next step is to construct the simplified film profile equation by replacing velocity terms into film thickness. Velocity component tangential to the flow direction, u, is reasonably approximated, satisfying the fully developed flow condition at far upstream region from the slide. This expression shows that the velocity profile is parabolic along the film thickness as following.

$$u=\frac{3}{h}\left\{\frac{y}{h}-\frac{1}{2}\left(\frac{y}{h}\right)^2\right\} \tag{6}$$

Velocity component normal to the flow direction, v, is also drawn by the continuity condition. Both velocity profiles satisfy the given boundary conditions (Eqs. 4a~5a)

$$v=\frac{y}{h}\left(\frac{dh}{dx}u\right) \tag{7}$$

A key step for deriving the film profile equation is to eliminate the pressure distribution using the integrated y-momentum equation (Eq. 2b) over the film thickness and the normal stress boundary condition. By substituting the pressure distribution and velocity profiles (Eqs. 6-7) into x-momentum equation (Eq. 2a) and then integrating it again over the film thickness, final film profile equation in the slide flow is accomplished.

$$\frac{1}{Ca}\frac{d}{dx}\left(\frac{(d^2h)/dx^2}{(1+(dh/dx)^2)^{3/2}}\right)+\left(\frac{6}{5}\frac{Re}{h^3}-St\cos\theta\right)\frac{dh}{dx}-\frac{3}{h^3}+St\sin\theta=0 \tag{8}$$

This third-order ordinary differential equation balances capillary pressure, liquid inertia, crosswise gravity, viscous shear, and streamwise gravity forces.

3. Simplified model of the curtain flow

The procedure to derive the simplified model for the curtain flow is very tedious and more difficult than the case of the slide flow, because the curtain sheet has both free surfaces, which can be deflected due to the different air pressures across the curtain. To understand the dynamics of free unsupported liquid curtain, sheet position or trajectory as well as the sheet thickness should be examined in curtain flow region. Under the several assumptions that curtain sheet thickness is very small compared to the length scale of flow direction, i.e. "thin sheet approximation," and shear stress is neglected in the curtain flow because this region is mainly governed by uniaxial extensional flow and deformation and velocity component normal to sheet trajectory, v, is very small, the shortened curvilinear two-dimensional (dimensionless) equations can be approximately arranged as shown below (Full generalized curvilinear Navier-Stokes equations were eloquently presented in Kistler (1983)). The configuration of curtain flow is portrayed in Fig. 2b.

Equation of continuity:

$$\frac{\partial u}{\partial s}+\frac{\partial v}{\partial n}=0 \tag{9}$$

Equation of s-momentum:

$$Re\left(u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial n}\right)=-\frac{\partial P}{\partial s}+St\sin\alpha+\frac{\partial\tau_{ss}}{\partial s} \tag{10}$$

where $\tau_{ss}=2\frac{\partial u}{\partial s}$

Equation of n-momentum:

$$-Reu^2\dot{\alpha}=-\frac{\partial P}{\partial n}-St\cos\alpha+\dot{\alpha}(\tau_{nn}-\tau_{ss})+\frac{\partial\tau_{nn}}{\partial n} \tag{11}$$

where $\dot{\alpha}=d\alpha/ds,\ \tau_{nn}=2\frac{\partial v}{\partial n}=-2\frac{\partial u}{\partial s}$

Normal stress condition at free surfaces:

$$-P_{\pm}+P_{a,\pm}+\tau_{nn,\pm}+\frac{1}{Ca}(2K_{\pm})=0 \quad \text{at } n=\pm H/2 \tag{12}$$

where $2K_{\pm}\equiv\kappa_{\pm}=\underline{n}\cdot\frac{d\underline{t}}{ds_f}=\pm\dot{\alpha}-\frac{\ddot{H}/2}{(1+(\dot{H}/2)^2)^{3/2}}=$ mean curvature of free surface.

(It is noted that last term in mean curvature is not eliminated to consider the effect of capillarity in s-momentum equation, though this is negligible compared with other terms under the given assumptions.)

Tangential stress condition at free surfaces:

$$\tau_{ns,\pm} = 0 \quad \text{at} \quad n = \pm H/2 \tag{13}$$

Kinematic condition at free surfaces:

$$u_{\pm} = \pm\frac{v_{\pm}}{\dot{H}/2} \quad \text{at} \quad n = \pm H/2 \tag{14}$$

where u and v denote velocity components parallel and normal to the curtain flow direction, s and n spatial coordinates parallel and normal to the flow direction, α inclination angle of mid-surface from the horizontal line, τ_{ij} extra stress components, H sheet thickness, and s_f arc length. Also subscripts + and – represent the top and bottom free surfaces, respectively, and overdots mean the derivatives with respect to s. Above dimensionless equations are measured through the same units as in the case of slide flow.

As explained in the slide flow, it is the key step to eliminate the pressure distribution in momentum equations using the following equality relation.

$$2P(n) = P(H/2) + P(-H/2) + \int_{H/2}^{n}\frac{\partial P}{\partial n}dn + \int_{-H/2}^{n}\frac{\partial P}{\partial n}dn \tag{15}$$

Using $\partial P/\partial n$ term of the n-momentum equation, the pressure field is given by

$$P(n) = \frac{P_{a,+}+P_{a,-}}{2} + \frac{1}{2Ca}(2K_{+}+2K_{-}) + Reu^2\dot{\alpha}n$$
$$-St\cos\alpha n + (\tau_{nn}-\tau_{ss})\dot{\alpha}n + \tau_{nn} \tag{16}$$

By integrating the n-momentum balance over the sheet thickness from $-H/2$ to $H/2$, approximate n-momentum equation representing the curtain trajectory is finally obtained.

$$-\dot{\alpha}\left(Reu^2H - 4\frac{du}{ds}H\right) + \Delta P + \frac{1}{Ca}(2\dot{\alpha}) + St\cos\alpha H = 0 \tag{17}$$

where, ΔP is the air pressure difference across the curtain. This equation balances crosswise inertia, viscous tensile, air pressure difference across the curtain, and crosswise gravity forces.

Also, integrating the s-momentum balance (Eq. 10) over the sheet thickness and inserting the pressure distribution (Eq. 16) lead to approximate s-momentum equation describing the sheet thickness along the curtain.

$$Re\frac{d}{ds}(u^2H) - \frac{H}{Ca}\frac{d}{ds}\left\{\frac{\ddot{H}/2}{(1+(\dot{H}/2)^2)^{3/2}}\right\} - \frac{d}{ds}\left(4\frac{du}{ds}H\right) - StH\sin\alpha = 0 \tag{18}$$

Above equation represents force balance of liquid inertia, capillary pressure, viscous tensile, and streamwise gravity. The velocity component, u, is substituted with the curtain thickness, H by the constant mass flow rate condition. From equations in curtain flow region, it can be demonstrated that the curtain sheet falls vertically, exactly agreeing with results presented by Brown (1961), when air pressures on both sides of the sheet are equal.

Above simplified models for both slide and curtain flows can be solved in finite difference approximation, i.e., 400 nodal points in each flow regime guaranteeing acceptable accuracy, by Newton iteration with the continuation scheme of operating parameters and with the boundary conditions that will be explained in next section.

4. Boundary conditions

To solve and match the profile equations of both slide and curtain flows derived in previous sections, several kinds of boundary conditions should be specified. In other words, upstream boundary condition in the slide flow region, the downstream boundary conditions in the curtain flow region, and finally matching conditions between two regions where curtain-forming region is roughly approximated. First, upstream boundary condition is sought from the asymptotic film profile by linearizing film profile equation, Eq. (8) without loss in accuracy, since the liquid film asymptotically approaches the fully-developed profile in far upstream region. Linearizing Eq. (8) with $h = h_0 + \varepsilon h_1$ (in this case, $h_0 = 1$), where ε represents tiny perturbation from h_0, yields

$$\frac{1}{Ca}\frac{d^3h_1}{dx^3} + \left(\frac{6Re}{5h_0^3} - St\cos\theta\right)\frac{dh_1}{dx} + \frac{9}{h_0^4}h_1 = 0 \tag{19}$$

The solution of this linearized equation is the combination of three exponential functions with exponents obtained from the algebraic cubic characteristic equation. Under the usual operating conditions of curtain coating process (i.e., larger slide inclination angle than 15^o and low capillary number), it has been proved that exponents have one negative real and one complex conjugate pair whose real part is positive. To guarantee uniform upstream flow on the slide, the complex conjugate set should be selected and thus makes the upstream asymptotic boundary condition on the slide have the standing wave form of film thickness at that regime.

$$\frac{d^2h}{dx^2} - 2\lambda_R\frac{dh}{dx} + (\lambda_R^2 + \lambda_I^2)(h - h_0) = 0 \tag{20}$$

where λ_R, λ_I denote the real and imaginary part of the conjugate complex set, respectively. Second, two downstream boundary conditions at the curtain flow describe the free-fall of the liquid curtain.

$$\frac{dH}{ds} = -\frac{St}{Re}\sin\alpha H^3 \quad \text{and} \quad \frac{d^2H}{ds^2} = -\frac{St}{Re}\left(3\sin\alpha H^2\frac{dH}{ds} + \cos\alpha\frac{d\alpha}{ds}H^3\right) \tag{21}$$

Otherwise, alternative of downstream boundary conditions can be chosen as constant take-up condition by moving substrate or web.

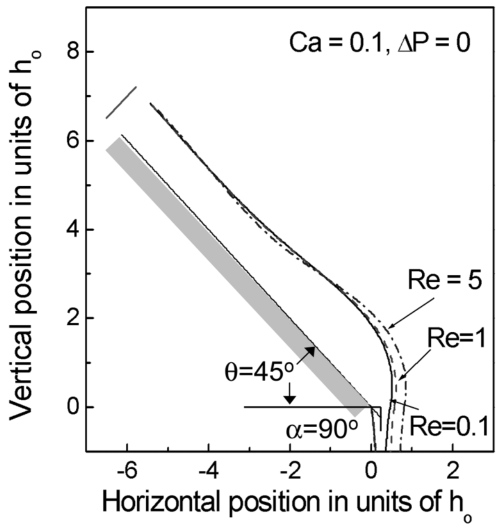
Third, boundary conditions near the curtain-forming region between the slide flow and the curtain flow regions should be also considered to smoothly connect the film thickness profiles from both regions. The separating line where both flow regions meet has been selected somewhat arbitrarily in this study. For simplicity, matching point is assumed so that the liquid falls vertically at the slide lip, though the point for keeping both momentum fluxes continuous may be determined. As matching conditions, the top free surface, surface inclination, and surface curvature must be continuous between the two regions.

5. Results and discussion

As mentioned above, the major objective of this study is to develop and test simplified models for curtain coating flows. Using these simplified models suggested in this study, effects of process conditions on the curtain profile and trajectory have been investigated. Fig. 3 depicts how inertia or Reynolds number affects the film profile. As Reynolds number rises, the upstream film profile on the slide flow becomes more slightly wavy caused by the characteristics of the asymptotic inflow boundary condition (Fig. 3a). In other words, asymptotic boundary condition includes standing wave properties of the liquid film, and this trend is more prominent when Reynolds number rises. Also, it can be seen from Fig. 3b that sheet profile become more and more thicker at the same position as Reynolds number increases under the given conditions.

The relative contribution of each force term on sheet profile is displayed in Fig. 4 under the specific operating conditions given in Fig. 3. In this case, gravity force is comparable to viscous shear in slide flow region and viscous tensile force and inertia force in the curtain flow region. Also, it is noted from this result that capillary pressure (or surface tension) is important only in the curtain-forming zone. Simplified model fails near the matching point, due to the disregard of complex flow behavior in the curtain forming region and also the different flow characteristics in both flow regimes, however, not significantly influencing film profile.

As another example, the effect of capillary number or surface tension on the sheet profile is portrayed in Fig. 5. High surface tension promotes standing wave of the liquid film on the upstream slide, and makes flow transition from slide to curtain near the curtain-forming region smooth.



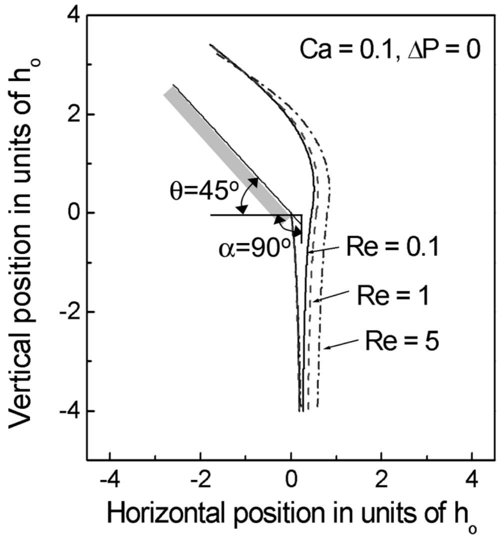


Fig. 3. The effect of inertia or Reynolds number on sheet profiles in (a) slide flow and (b) curtain flow.

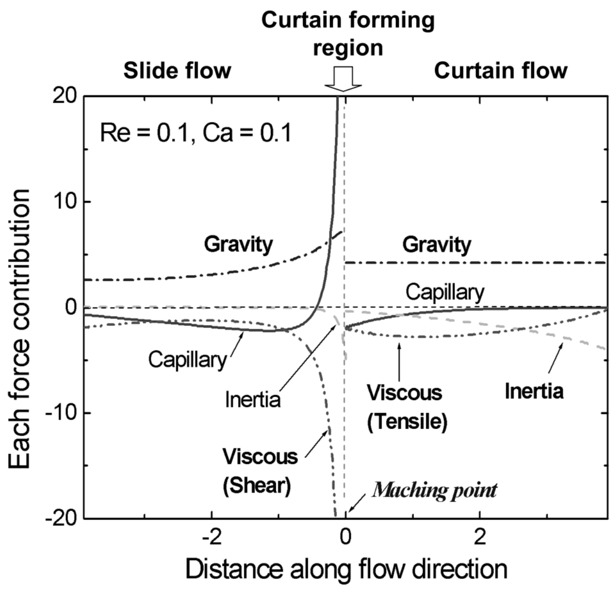


Fig. 4. The relative contribution of force balances in the simplified models.

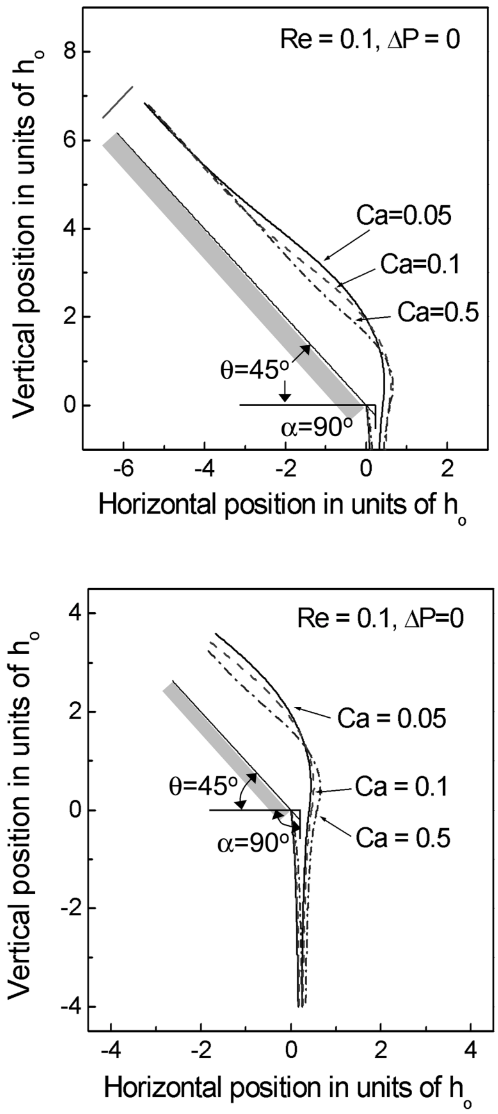


Fig. 5. The effect of surface tension or capillary number on sheet profiles in (a) slide flow and (b) curtain flow.

One interesting result is to show the effect of air pressure difference across the curtain on the sheet profile (Fig. 6). As explained above, in the case of ΔP equal to zero, liquid sheet falls vertically without being affected by any circumstances. However, when ΔP rises, curtain trajectory is more deflected, maintaining the same film thickness profile. This fact proves that the stability or sensitivity of liquid curtain can be critically affected by unexpected external disturbances such as pressure difference of the surrounding air.

The more detailed dynamics in the curtain flow region and its stability and sensitivity results with respect to the specified disturbances, for example, air pressure difference, will be scrutinized further using time-dependent one-dimensional governing equations in Jung and Scriven (2004).

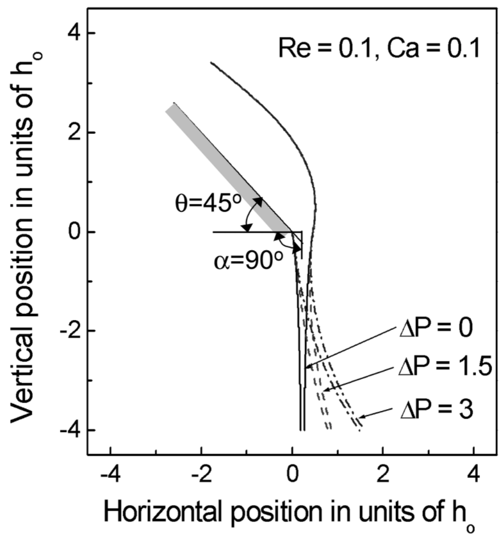


Fig. 6. The effect of pressure difference across the curtain on sheet profiles.

6. Conclusions

Curtain coating flow has been investigated using the simplified models newly developed in this study. The approximate governing equations for both slide and curtain flows have been successfully derived by thin film approximation and the integral momentum balance approach of Higgins and Scriven (1979) and Kistler (1983). These models draw on the sheet profile equations for curtain thickness and its trajectory in the curtain flow as well as the film profile equation in the slide flow. The equations have been solved in finite difference method by Newton iteration with continuation. The results by the simplified models, qualitatively agreeing with full theory and experimental observations, show that how process conditions such as inertia (Reynolds number), surface tension (capillary number), inclination angle of the slide, and air pressure difference across the sheet affect sheet trajectory and thickness profile. More detailed dynamics in the curtain flow and its stability and sensitivity analyses will be examined further.

Acknowledgements

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